Approximation of the Hilbert Transform on the unit circle

Luisa Fermo and Valerio Loi

Department of Mathematics and Computer Science, University of Cagliari, Via Ospedale 72, 09124 Cagliari, Italy
fermo@unica.it, v.loi18@studenti.unica.it

Abstract

In this poster, we present a numerical procedure to approximate the Hilbert transform [2]

$$(Hf)(z) = \frac{1}{\pi} \int\!\!\!\!\!\!- _{\Gamma} \frac{f(t)}{t-z} dt, \qquad z \in \Gamma$$

where the integral is understood as the Cauchy principal value sense, Γ is the unit circle in $\mathbb{C}$

$$\Gamma = e^{\mathrm{i}\mathbb{T}} = \{z \in \mathbb{C} : |z| = 1\}, \qquad \mathbb{T} = [-\pi, \pi],$$

and $f : \Gamma \to \mathbb{C}$ is a known function.

The method takes advantage of the Szegö and anti-Szegö quadrature rules [1]. Numerical examples, showing the accuracy of the proposed approach, will be given.

References

1. S.M. Kim and L. Reichel. Anti-Szegö quadrature rules. *Mathematics of Computation*, 76(258):795–810, 2007.
2. N.I. Muskhelishvili. *Singular integral equations*. Dover publications, 1992.