Riesz representers as a basis for regularized solution of first kind integral equations

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Overdetermined systems of first kind integral equations arise in many applications. It is well-known that Fredholm integral equations of the first kind are often ill-posed problems. When the right-hand side is discretized, e.g., when it consists of experimental measurements, the difficulties related to ill-posedness are enforced, as the problem admits infinitely many solutions. We propose a numerical method to compute the minimal-norm solution of a system of the form

$$\begin{cases} \displaystyle\int_a^b k_\ell(x_{\ell,i}, t)\, f(t)\, dt = g_\ell(x_{\ell,i}), & \ell = 1, \dots, m, \quad i = 1, \dots, n_\ell, \\ f(a) = f_0, \ f(b) = f_1, \end{cases}$$

in the presence of boundary constraints. The algorithm stems from the Riesz representation theorem and operates in a reproducing kernel Hilbert space (RKHS). Since the resulting linear system is strongly ill-conditioned, we construct a regularization method based on a truncated expansion of the minimal-norm solution in terms of the singular functions of the integral operator defining the problem. Numerical experiments, both artificial and deriving from an application in applied geophysics, are presented to show that the new method is extremely effective when the sought solution is smooth, but produces significant information even for non-smooth solutions; [1], [2].

References

[1] P. Díaz de Alba, L. Fermo, F. Pes, and G. Rodriguez: Minimal-norm RKHS solution of an integral model in geo-electromagnetism, In: 21st International Conference on Computational Science and Its Applications (ICCSA) (2021) 21–28.

[2] P. Díaz de Alba, L. Fermo, F. Pes, and G. Rodriguez: Regularized minimal-norm solution of an overdetermined system of first kind integral equations, Numer. Algorithms (2022) https://doi.org/10.1007/s11075-022-01282-2.