

# ERGODICITY COEFFICIENTS FOR SECOND-ORDER MARKOV CHAINS

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We extend a class of ergodicity coefficients [1] from the matrix case to the higher-order setting of nonnegative stochastic tensors of order three, denoted hereafter with a bold letter  $\mathbf{P}$ . Similarly to the matrix case, the new higher-order ergodicity coefficients provide novel conditions that guarantee the existence and uniqueness of a positive Z-eigenvector of  $\mathbf{P}$  corresponding to the eigenvalue one, i.e., a vector  $x$  such that  $\mathbf{P}xx = x$ . Moreover, they allow us to prove new conditions for the global convergence of the so-called higher-order and alternate higher-order power methods, defined by  $x_{k+1} = \mathbf{P}x_kx_k$  and  $x_{k+1} = \mathbf{P}x_kx_{k-1}$ , respectively. Example applications include the analysis of the behaviour of second-order Markov chains, such as the multilinear PageRank [2], and the convergence of the shifted higher-order power method [3].

## References

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