ON THE IDENTIFICATION OF THE REGULARIZATION PARAMETER IN ILL-POSED PROBLEMS

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We are concerned with the solution of discrete ill-posed problems of the form

$$\min_{\mathbf{x}\in\mathbb{R}^n} \|A\boldsymbol{x} - \boldsymbol{b}\|, \tag{1}$$

where $A \in \mathbb{R}^{m\times n}$ is a large matrix whose singular values decay gradually to zero without a significant gap.

A good approximation of the solution can often be computed by first replacing the least-squares problem (1) by a nearby problem, that is regularizing. Here we will discuss two of the most popular regularization methods, namely the Tikhonov regularization method and the Truncated Singular Value Decomposition (TSVD).

When an accurate bound for the norm of the error $\|\boldsymbol{e}\|$ is available, a suitable value of the regularization parameter can often be determined with the aid of the discrepancy principle. However, for many discrete ill-posed problems (1), such a bound is not known. Here we discuss the use of the Generalized Cross Validation (GCV) in the case of Tikhonov regularization. This method requires the minimization of the GCV function. We will present two fairly inexpensive ways to determine bounds for the GCV function for large matrices A [1, 2].

TSVD method allows one to replace the ill-conditioned matrix A by a well-conditioned low-rank matrix obtained by keeping the first k singular triplets of its Singular Value Decomposition. We will present a method to identify the regularization parameter k when the TSVD is used as a regularization method based on an extrapolation procedure.

References

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