ANALYSIS OF BLOCK KRYLOV SUBSPACE METHODS RELYING ON GENERAL BLOCK INNER PRODUCTS

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Block Krylov subspace methods for solving s simultaneous linear systems

$$A\mathbf{X} = \mathbf{B}, \quad \text{where } A \in \mathbb{C}^{n\times n}, \quad \mathbf{B} = [\mathbf{b}_1|\cdots|\mathbf{b}_s] \in \mathbb{C}^{n\times s}$$

can be faster than methods that treat individually the systems $A\mathbf{x}_i = \mathbf{b}_i$, $i = 1,\ldots,s$, for two reasons: Since a block Krylov subspace is larger than any of the individual subspaces, one can extract more accurate approximations for the same total investment of matrix-vector multiplications. And since the multiplication of A with a block vector $\mathbf{B}$ can be implemented more efficiently than s individual multiplications, they require less memory access and allow for batch communication.

Starting from the block FOM method, we develop a general concept of modified block FOM methods which includes block GMRES and a "Radau-Arnoldi" variant. We present results on variational characterizations, on properties of the underlying residual matrix polynomials and on comparisons between the iterates for different block inner products. Particular emphasis will then be put on shifted families of block linear systems of the form

$$(A + tI)\mathbf{X}_t = \mathbf{B}, \ \ t \in T \subseteq \mathbb{C},$$

where we discuss a computational strategy to keep block residuals "co-spatial" for all t in the presence of restarts. This allows to build the block Krylov subspace only once for all shifts t, even after a restart. We show to which extent results from the non-block case do have a natural counterpart for the block case. Maintaining co-spatiality in restarts is mandatory for being able to express the error when approximating matrix functions via block Krylov subspaces, a topic which will be investigated in depth in Kathryn Lund's contributed talk.