Rank structure based solvers for 2D fractional diffusion equations

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In this work, we consider the discretization of time-space diffusion equations with fractional derivatives in space and either 1D or 2D spatial domains. The use of implicit Euler scheme in time and finite differences or finite elements in space leads to a sequence of dense large scale linear systems describing the behavior of the solution over a time interval. We prove that the coefficient matrices arising in the 1D context are rank structured and can be efficiently represented using hierarchical matrices (HODLR format). Qualitative and quantitative estimates for the rank of the off-diagonal blocks of these matrices are presented. Their rank structure is then leveraged to design fast solvers for problems with 2D spatial domains that can be reformulated as matrix equations. In detail, when the right-hand side of the fractional diffusion problem is regular or sparse, the known term of the matrix equation has low-rank properties. This enables the use of Krylov subspace methods which combined with the technology of hierarchically rank structured matrices yields a lower computational complexity in comparison with the current state of the art techniques.