A COMPARISON OF REGULARIZATION METHODS FOR SOLVING NONLINEAR PROBLEMS

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Let us assume that $F(\mathbf{x})$ is a nonlinear Fréchet differentiable function, with value in $\mathbb{R}^m$ for any $\mathbf{x} \in \mathbb{R}^n$. For a given $\mathbf{b} \in \mathbb{R}^m$, we solve the least squares problem $\min_{\mathbf{x}} \|\mathbf{r}(\mathbf{x})\|^2$, where $\mathbf{r}(\mathbf{x}) = F(\mathbf{x}) - \mathbf{b}$ is the residual vector function, by applying both Newton's and Gauss–Newton methods [1].

The nonlinear function $F(\mathbf{x})$ is considered ill-conditioned in a domain $\mathcal{D}$, when the condition number $\kappa(J)$ of the Jacobian $J = J(\mathbf{x})$ of $F(\mathbf{x})$ is large for any $\mathbf{x} \in \mathcal{D}$. It may also happen that, during the iteration of Gauss–Newton method, the matrix J becomes rank-deficient. Under this assumption, it is common to apply a regularization method to each step of the Gauss–Newton method. We compare this situation to applying the same regularization method to the initial nonlinear least squares problem. We apply these two approaches to a geophysical model used for electromagnetic data inversion [2, 3].

References

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