SIMULTANEOUS GAUSS QUADRATURE

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Suppose $f : \mathbb{R} \to \mathbb{R}$ is a given function and $\mu_1, \ldots, \mu_r$ are positive measure on the real line. The goal is to approximate the r integrals $\int f(x)\, d\mu_j(x)$, $1 \leq j \leq r$, by sums of the form $\sum_{k=1}^{N} f(x_k)\lambda_k^{(j)}$, $1 \leq j \leq r$, using the same quadrature nodes $\{x_j, 1 \leq j \leq N\}$ but with quadrature weights $\{\lambda_k^{(j)}, 1 \leq k \leq N\}$ depending on the measure μ_j. Similar to Gaussian quadrature, there is an optimal choice for the quadrature nodes that maximizes the degree of accuracy: one needs to take the zeros of a multiple orthogonal polynomial for the measures $(\mu_1, \ldots, \mu_r)$. I will give properties of the quadrature nodes and the quadrature weights for two cases. First I will deal with $r = 2$ and μ_1 and μ_2 positive measures with support on two disjoint intervals [1]; the second case is $r = 3$ and the measures are normal weights with means $-c, 0, c$ with c sufficiently large [2]. In these cases the quadrature nodes belong to r disjoint intervals $\Delta_1, \ldots, \Delta_r$ and the quadrature weights $\lambda_k^{(j)}$ are positive for the nodes on Δ_j, but alternate in sign for the other nodes. These nodes with alternating sign, however, are exponentially small and hence can be ignored in practice.

References

[1] D.S. Lubinsky, W. Van Assche, *Simultaneous Gaussian quadrature for Angelesco systems*, Jaén J. Approx. 8 (2016), 113–149.

[2] W. Van Assche, A. Vuerinckx, *Multiple Hermite polynomials and simultaneous Gaussian quadrature*, ETNA 50 (2018), 182–198.