

A NEW APPROACH TO CONJUGATE GRADIENT AND GMRES CONVERGENCE

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Known as one of the best iterative methods for solving symmetric positive definite linear systems, CG generates as FOM an Hessenberg matrix which is symmetric then triangular. This specific structure may be really helpful to understand how does behave the convergence of the conjugate gradient method and its study gives an interesting alternative to Chebyshev polynomials. The talk deals about some new bounds on residual norms and error A -norms using essentially the condition number.

GMRES is one of the most widely used iterative methods for the solution of linear system of equations, with a large real or complex nonsingular matrix. Convergence properties of GMRES are discussed by many authors. But most convergence results are obtained as a polynomial approximation problem.

We will show that bounding the norm of the residual vectors determined by GMRES in terms of the eigenvalues of the matrix, is a difficult constrained optimization problem. We therefore focus on diagonalizable matrices and in the particular case where the matrix is we will show how to derive a bound of the norm of the residual by solving a constrained optimization problem using Lagrange multipliers.