Generalized exponential integrators for fractional differential equations

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Exponential integrators are powerful and well–established methods particularly suited for the time–integration of semilinear systems of ordinary differential equations (ODEs) with linear stiffness. By solving exactly the stiff term, exponential integrators allow to integrate the remaining non–stiff part of the system by means of explicit schemes without calling for severe restrictions on the step-size.

In this talk we discuss the generalization of exponential integrators to problems of non integer orders, namely fractional differential equations (FDEs), which are nowadays used in several areas, including biology, finance, physics and control theory, to model systems exhibiting anomalous dynamics [1].

The generalization of exponential integrators to FDEs presents some challenges: indeed, a more difficult function, specifically a Mittag-Leffler type function, has to be evaluated with matrix arguments; furthermore the presence of a persistent memory (a typical feature of FDEs) demands for different and more expensive techniques for the time–discretization [2].

In this talk we discuss the main computational issues and we present a class of exponential integrators for FDEs. Some results on accuracy and stability are also studied and we show, by means of some numerical experiments, the effectiveness of the proposed approach.

References

[1] K. Diethelm, *The Analysis of Fractional Differential Equations*, LNM Vol. 2004, Springer 2010.

[2] R. Garrappa and M. Popolizio, *Generalized Exponential Time Differencing methods for fractional order problems*, submitted (2010).