Lezione 6

14/10/24

EQUAZIONI E DISEQUAZIONI DI II GRADO

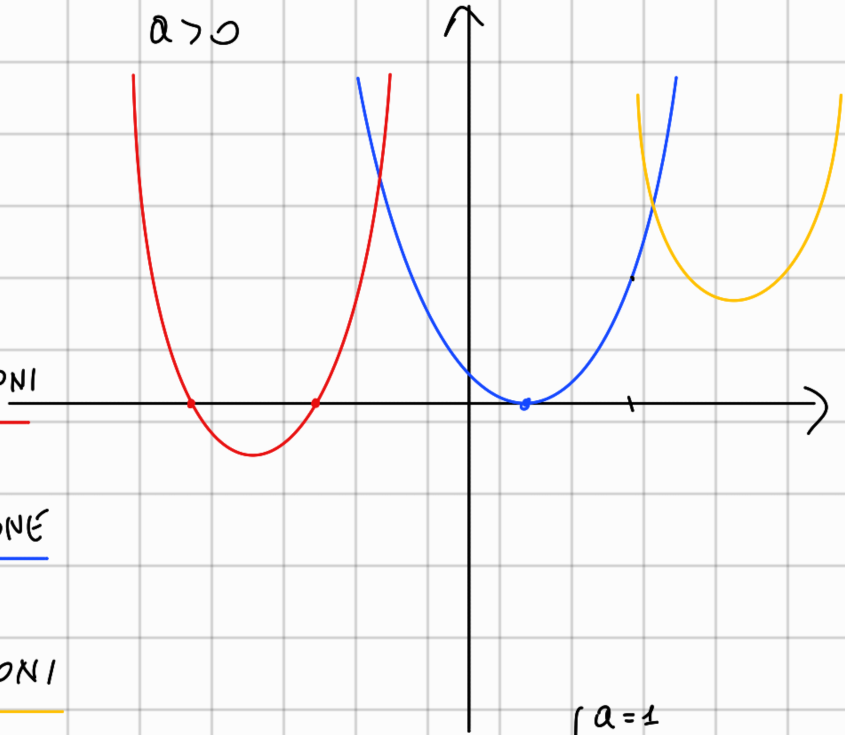
$$ax^2+bx+c=0$$

$$\Delta = b^2-4ac$$

SE $\Delta > 0$ 2 SOLUZIONI

$\Delta = 0$ 1 SOLUZIONE

$\Delta < 0$ 0 SOLUZIONI



$$x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a}$$

ESEMPIO:

$$y = x^2 \to \begin{cases} a=1 \\ b=0 \\ c=0 \end{cases}$$

$$\Delta = 0^2 - 4 \cdot 1 \cdot 0 = 0^2 - 0 = 0$$

$$x_{12} = \frac{-0 \pm \sqrt{0}}{2 \cdot 1} = 0$$

INTERPRETAZIONE DEI COEFFICIENTI $y = ax^2+bx+c$

$a,b,c \in \mathbb{R}$

$a \neq 0$ → $a > 0$

→ $a < 0$

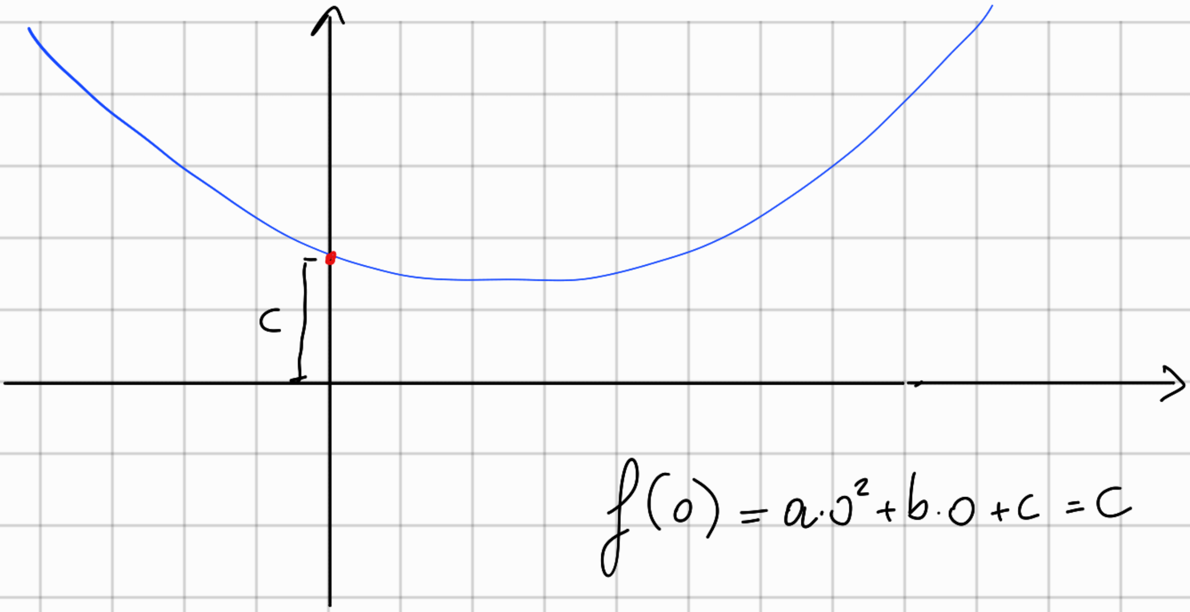
Più |a| è grande più la parabola sarà "stretta".

|a| vicino a 0 indice una parabola più "aperta".

C

TRASLAZIONE

"ALTO - BASSO"



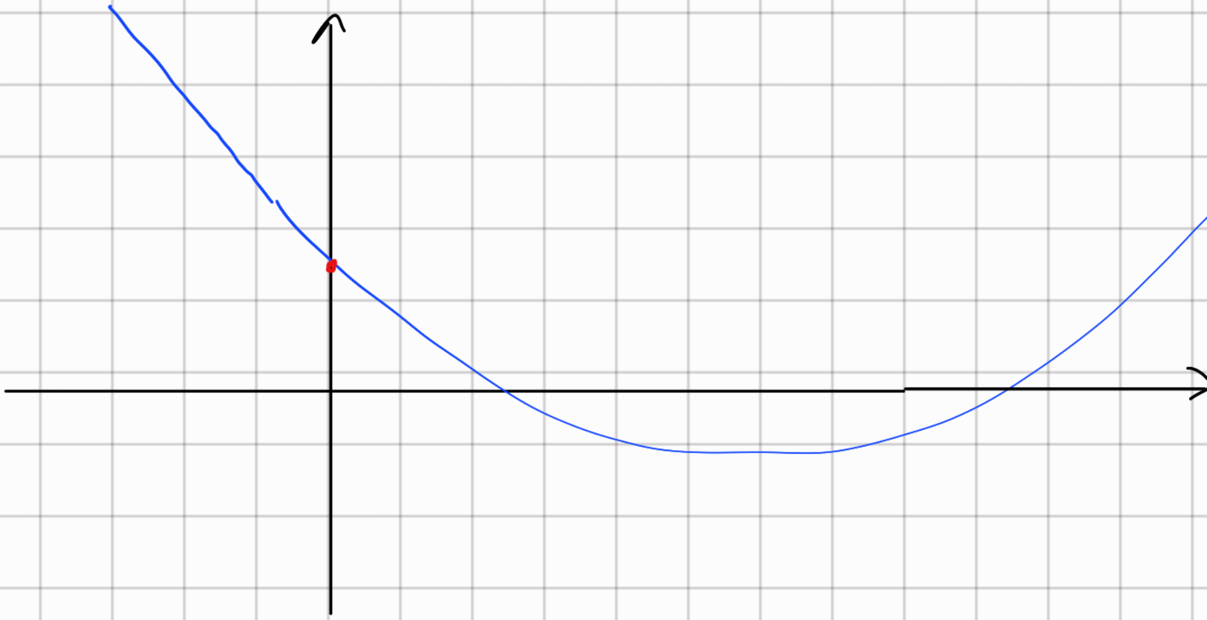
$f(0) = a \cdot 0^2 + b \cdot 0 + c = c$

b

TRASLAZIONE

"DESTRA - SINISTRA"

(RISPETTANDO C)



ESEMPIO

Risolvere l'equazione $\boxed{x^2 + x - 6 = 0}$

$\Delta = 1 - 4 \cdot 1 \cdot (-6) = 1 + 24 = 25$

$b^2 - 4ac$

$\sqrt{25} = 5$

$x_{1,2} = \dfrac{-1 \pm 5}{2}$ → $x_1 = \dfrac{-1-5}{2} = -3$ → $x_2 = \dfrac{-1+5}{2} = 2$

$S = \{-3, 2\}$

SCOMPOSIZIONE

$$\boxed{ax^2 + bx + c = a(x - x_1)(x - x_2)}$$

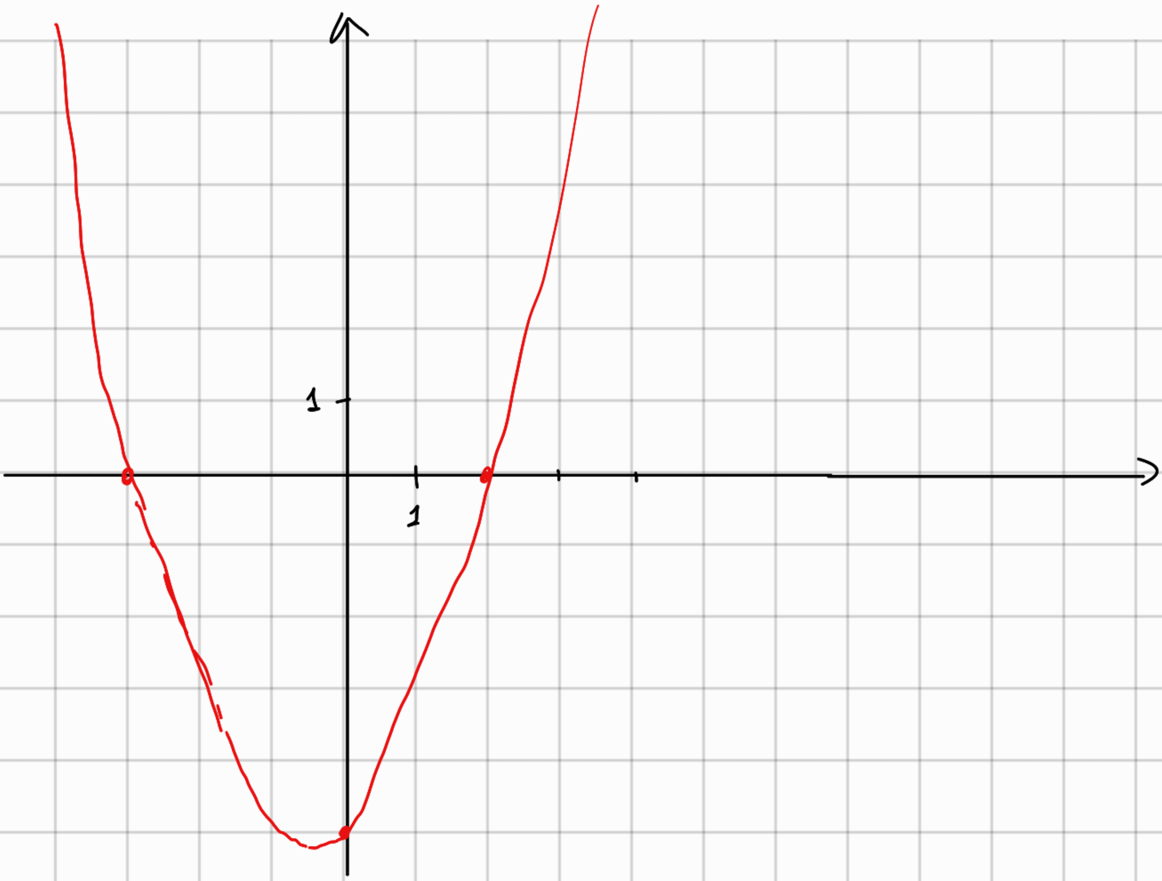
DIMOSTRARE

(SOLUZIONE IN FONDO)

$(x+3)(x-2)$

$x^2 + 3x - 2x - 6$

$x^2 + x - 6$



DISEQUAZIONI

$$x^2+x-6>0$$

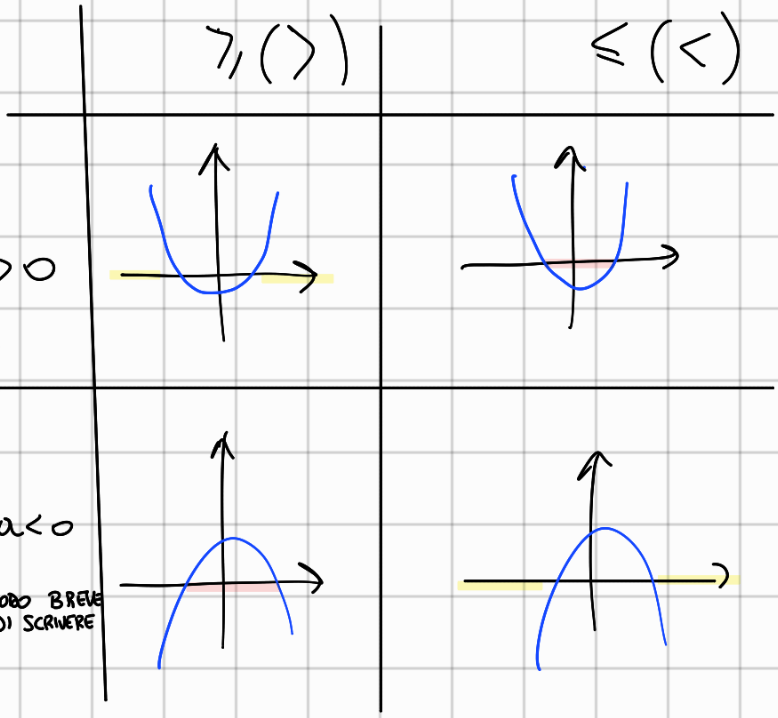
$$S=\{x : x<-3 \lor x>2\}$$

PER INTERVALLI $(-\infty,-3)\cup(2,+\infty)$

CASO 1 $\boxed{\Delta>0}$

Calcolo x_1, x_2

a>0



a<0

ESTERNI $S=\{x\in\mathbb{R} : x\le x_1 \lor x>x_2\}$

$= (-\infty,x_1)\cup(x_2,+\infty)\subseteq\mathbb{R}$

INTERNO $S=\{x\in\mathbb{R} : x_1\le x\le x_2\}$

MODO BREVE DI SCRIVERE

$x>x_1 \land x<x_2$

$= [x_1,x_2]$

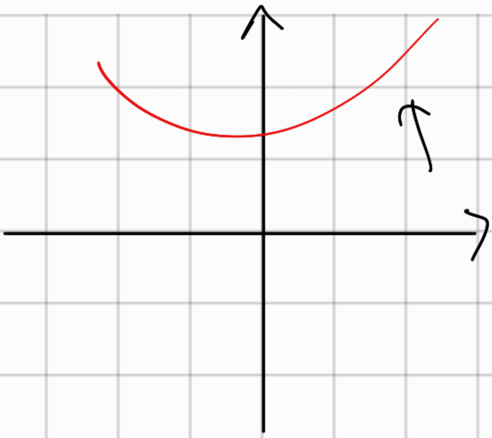
CASO 2 $\boxed{\Delta<0}$

↓

SE a e verso sono concordi

↓

$S=\mathbb{R}$



$a>0$

$a<0$

a, verso sono discordi $S=\emptyset$

CASO 3 $\boxed{\Delta=0}$, si calcola $x_1=x_2$, poi

	$>$	$\geq$	$<$	$\leq$
$a>0$	$S=\mathbb{R}\setminus\{x_1\}$	$S=\mathbb{R}$	$S=\emptyset$	$S=\{x_1\}$
$a<0$	$S=\emptyset$	$S=\{x_1\}$	$S=\mathbb{R}\setminus\{x_1\}$	$S=\mathbb{R}$

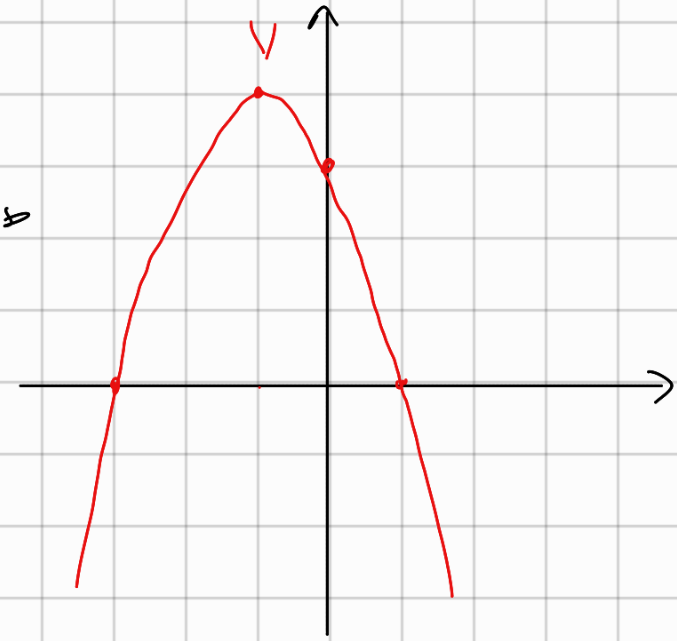
↓ SI SCRIVE ANCHE COSÌ

$S=(-\infty,x_1)\cup(x_1,+\infty)$

ESEMPIO: TRACCIARE IL GRAFICO

$$f(x) = -x^2 - 2x + 3 \qquad \mathbb{R} \to \mathbb{R}$$

- $a < 0 \Rightarrow$ CONCAVITÀ VERSO IL BASSO
- $c = 3$



$$-x^2 - 2x + 3 = 0$$

$$\Downarrow$$

$$x^2 + 2x - 3 = 0$$

$$\Delta = 4 + 12 = 16$$

$$x_{12} = \frac{-2 \pm 4}{2} \quad x_1 = \frac{-2-4}{2} = -3 \qquad x_2 = \frac{-2+4}{2} = 1$$

$$V = \left(-\frac{b}{2a}, -\frac{\Delta}{4a}\right) = \left(-\frac{-2}{-2}, -\frac{16}{-4}\right) = (-1, 4)$$

$$-x^2 - 2x + 3 = 0$$

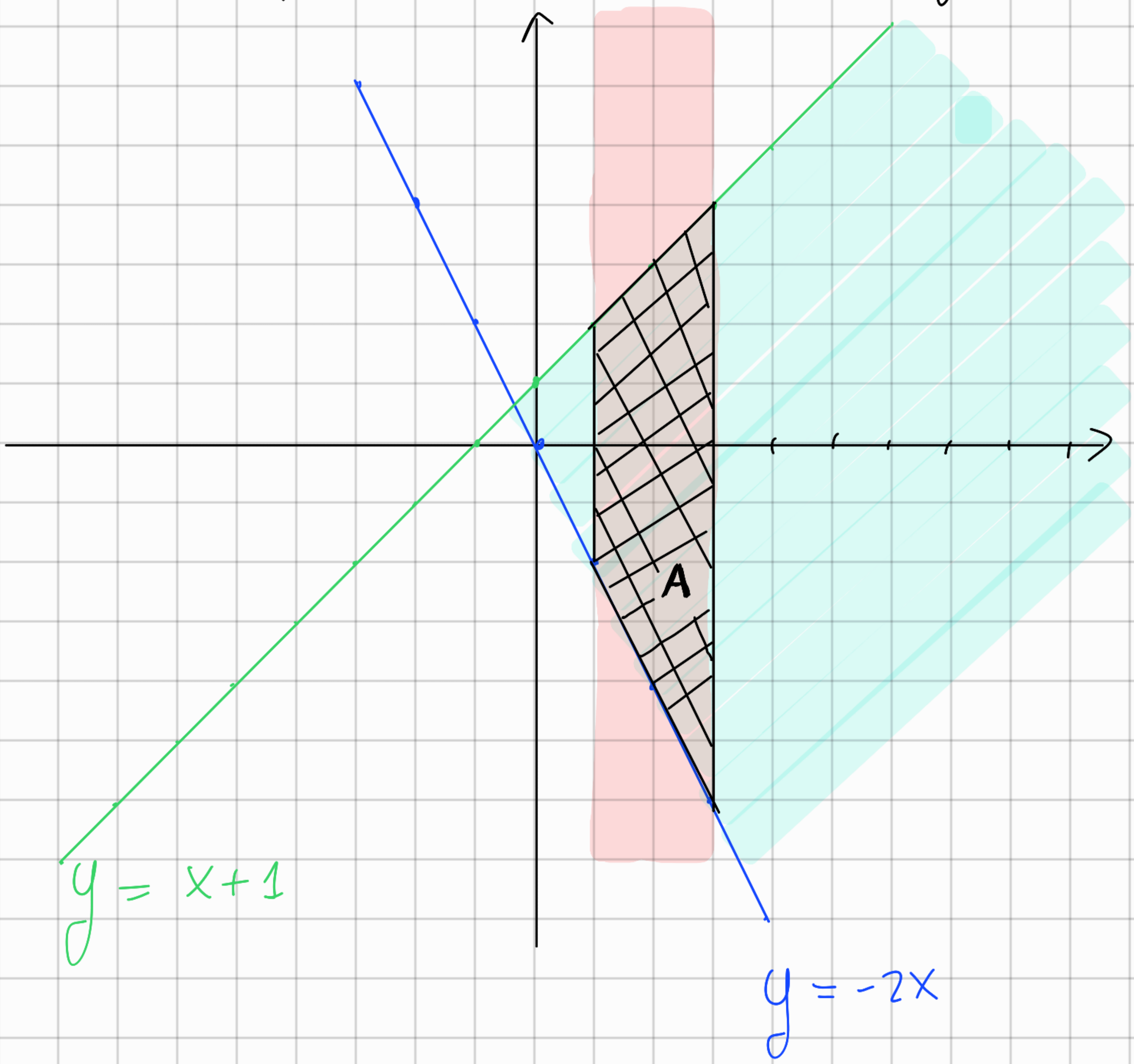
$$\Delta = 4 + 12 = 16$$

$$x_{12} = \frac{2 \pm 4}{-2} \quad x_1 = \frac{2+4}{-2} = -3 \qquad x_2 = \frac{2-4}{-2} = 1$$

ESERCIZIO

DISEGNARE L'INSIEME

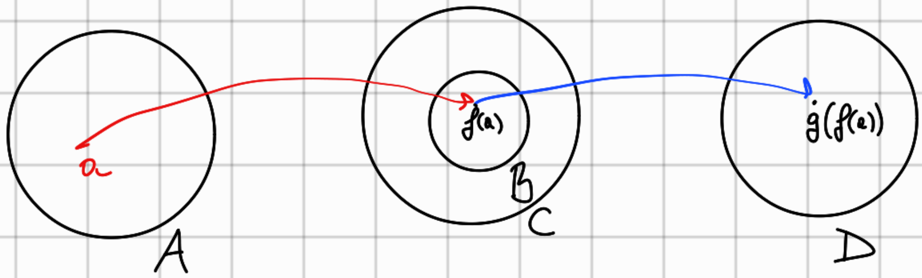
$$A = \{(x,y) \in \mathbb{R}^2 : \underbrace{1 \le x \le 3} \wedge -2x \le y \le x+1\}$$



DEFINIZIONE

Siano $f: A \to B$ $\quad g: C \to D$ funzioni

dove $B \subseteq C$



Allora

la $\boxed{\text{COMPOSIZIONE}}$ tra le due funzioni $g \circ f$ è la funzione

$$\begin{aligned} A &\longrightarrow D \\ a &\longmapsto g(f(a)) \end{aligned}$$

$$f(x) = x^2+1$$

$$\begin{aligned} \mathbb{R} &\longrightarrow \mathbb{R} \longrightarrow \mathbb{R} \\ x &\longmapsto x^2 \longmapsto x^2+1 \end{aligned}$$

Disegnare $f(x) = |x+2|$

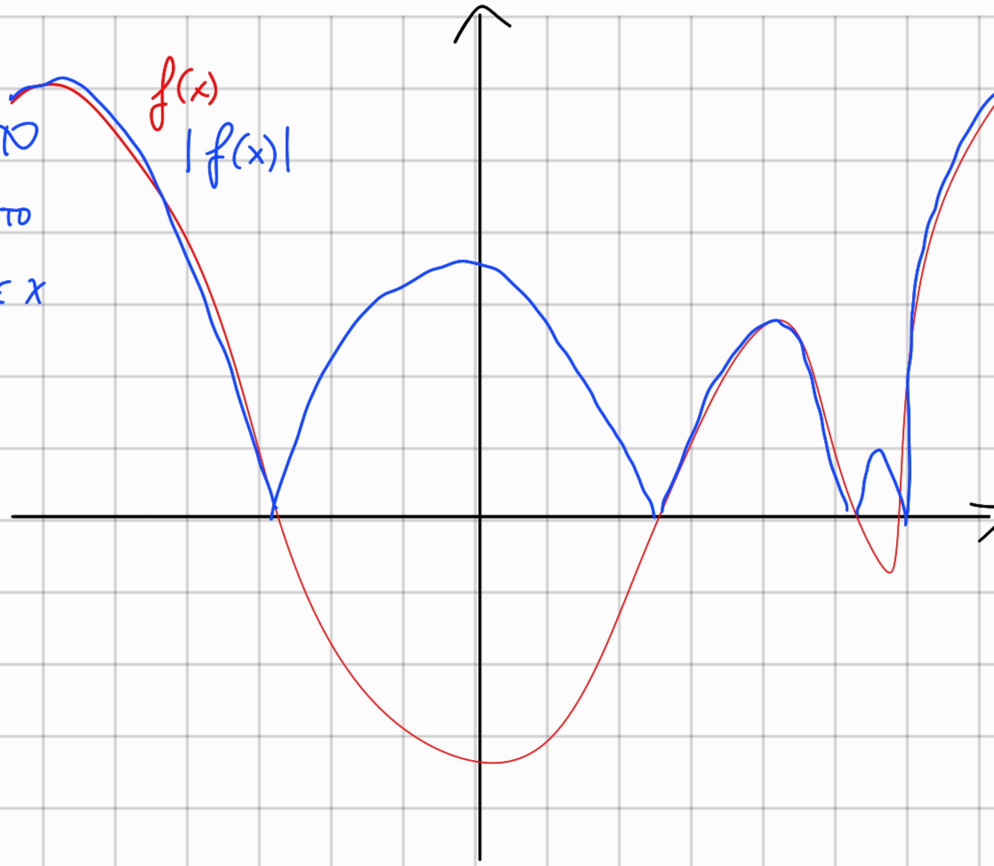
$$g: \mathbb{R} \longrightarrow \mathbb{R} \qquad h: \mathbb{R} \longrightarrow \mathbb{R}$$

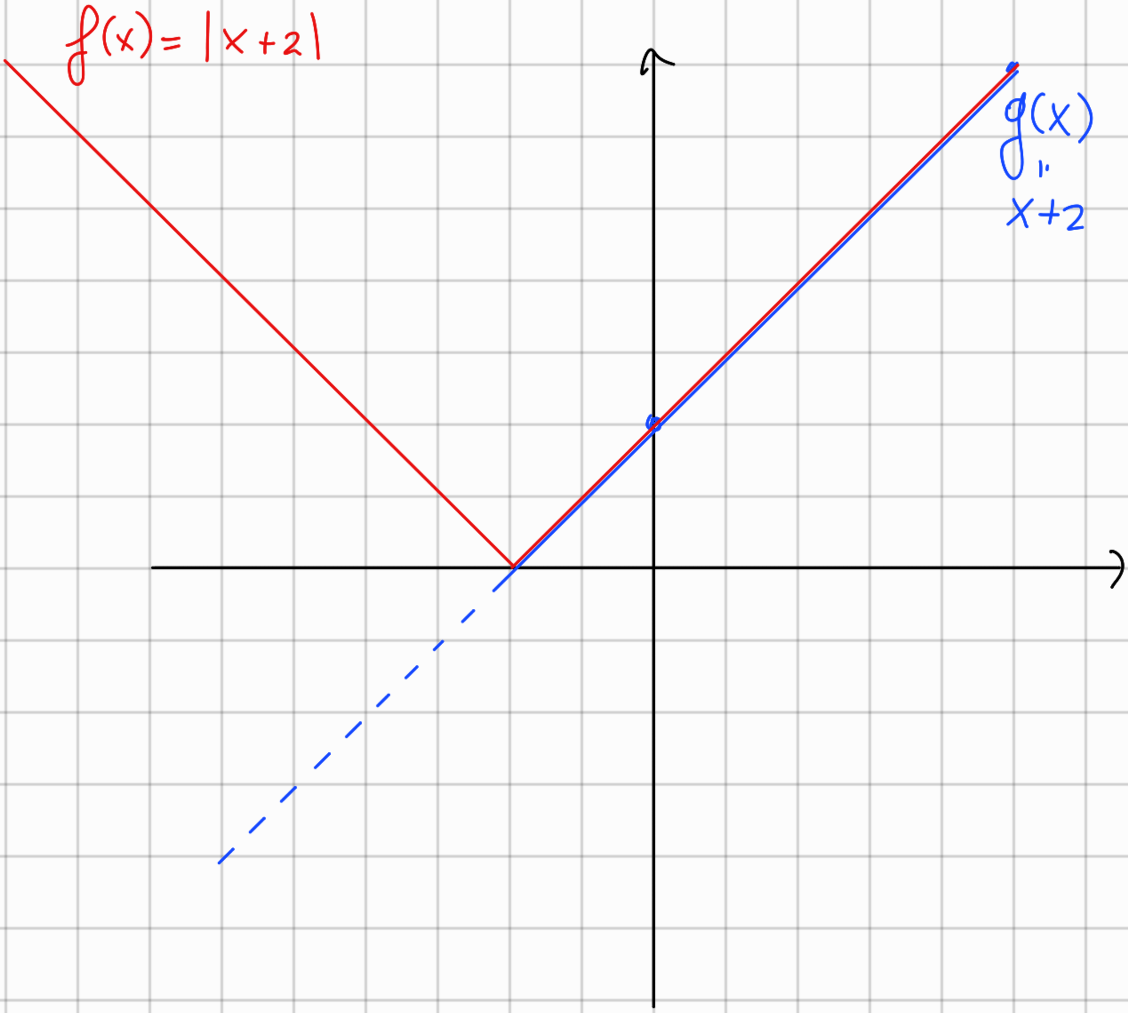
$$x \longmapsto x+2 \qquad x \longmapsto |x|$$

$g \circ h$ $\qquad \boxed{h \circ g}$ $\quad h(g(x)) = h(x+2)$

$= |x+2|$

GRAFICAMENTE
FARE IL VALORE ASSOLUTO
SIGNIFICA SPECCHIARE TUTTO
CIÒ CHE SI TROVA SOTTO L'ASSE X





PER ESERCIZIO

- $|x^2 - 4|$
- $|x^2 + 4x| + 1$

DISEQUAZIONI FRATTE

$(\leq, >, <)$

$$\frac{f(x)\,g(x)}{h(x)} \geq 0$$

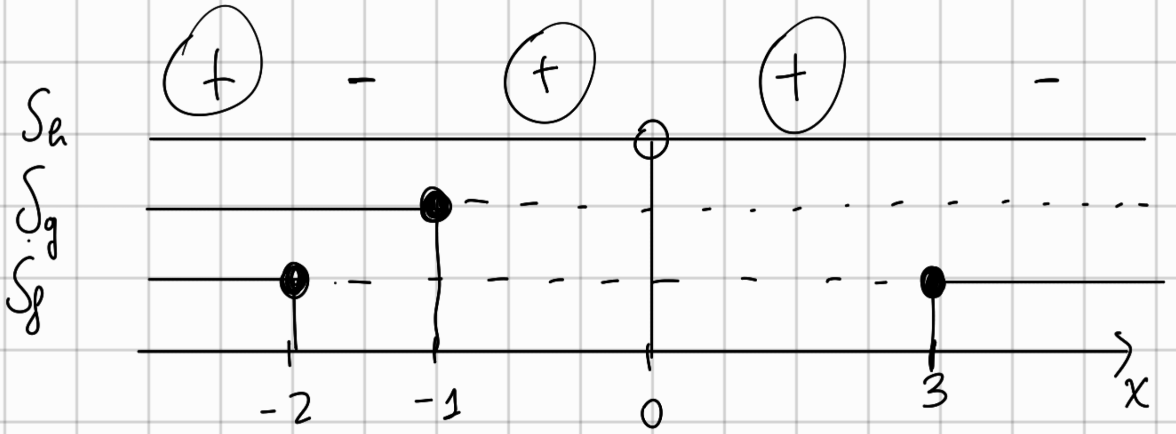
SI STUDIANO SEPARATAMENTE

NON $\geq$

$f(x) \geq 0$ | $g(x) \geq 0$ | $h(x) > 0$

$x \leq -2 \vee x \geq 3$

$S_f = (-\infty, -2] \cup [3, +\infty)$ $\quad S_g = x \leq -1$ $\quad S_h = \mathbb{R} \setminus \{0\}$



$[-1 \leq x < 0 \vee 0 < x \leq 3]$

$$S = \{x \in \mathbb{R} : [x \leq -2] \vee [-1 \leq x \leq 3 \wedge x \neq 0]\}$$

$(-\infty, -2] \cup [-1, 0) \cup (0, 3]$

PER CASA

- $\dfrac{x^2+25}{x^2-4x} < 0$ $\qquad S = (0,4)$

- $\dfrac{x^2+2x-5}{x^2-6x+8} \leq 0$ $\qquad S = [-1-\sqrt{6}, -1+\sqrt{6}] \cup (2,4)$

- $\dfrac{x^2-2}{2x-3} \geq 1$ $\qquad S = \{1\} \cup \left(\frac{3}{2}, +\infty\right)$

- $\dfrac{3}{x-2} < \dfrac{2x}{3+x}$ $\qquad S = (-\infty, -3) \cup (-1, 2) \cup \left(\frac{9}{2}, +\infty\right)$

DIMOSTRAZIONE SCOMPOSIZIONE POLINOMI 2° GRADO

Voglio fare vedere che $ax^2+bx+c = a(x-x_1)(x-x_2)$
dove
$$x_{12} = \frac{-b \pm \sqrt{\Delta}}{2a} \quad , \quad \Delta = b^2-4ac$$

$\Rightarrow$

$$a\,(x-x_1)(x-x_2) =$$

$$a\left[x - \left(\frac{-b-\sqrt{\Delta}}{2a}\right)\right]\left[x - \left(\frac{-b+\sqrt{\Delta}}{2a}\right)\right] =$$

$$a\left[x^2 - \frac{-b-\sqrt{\Delta}}{2a}x - \frac{-b+\sqrt{\Delta}}{2a}x + \left(\frac{-b-\sqrt{\Delta}}{2a}\right)\left(\frac{-b+\sqrt{\Delta}}{2a}\right)\right] =$$

$$ax^2 - \left[\frac{-b\cancel{-\sqrt{\Delta}}-b\cancel{+\sqrt{\Delta}}}{2}\right]x + \frac{b^2\cancel{+\sqrt{\Delta}b}\cancel{-\sqrt{\Delta}b}-\Delta}{4a} =$$

$$ax^2 - \frac{-\cancel{2}b}{\cancel{2}}x + \frac{\cancel{b^2}-\cancel{b^2}+4ac}{4a} =$$

$$ax^2+bx+c$$

$\square$