Lezione 10

23/10/24

Risolvere

$$\boxed{\log_a(x) = y} \iff \boxed{a^y = x}$$

- $2^x = \frac{1}{8}$

$\frac{1}{8} = \frac{1}{2^3} = 2^{-3}$

MODO 1

$$\log_2\left(\frac{1}{8}\right) = x$$

$$\log_2\left(2^{-3}\right) = x$$

$$-3 = x$$

MODO 2

$$2^x = 2^{-3}$$

$\downarrow \leftarrow$ INIETTIVITÀ DELL'ESPONENZIALE

$$x = -3$$

- $10^x = \frac{1}{3}$

$$\downarrow$$

$$x = \log_{10}\left(\frac{1}{3}\right)$$

- $(2\sqrt{3})^x = 144$

CON LE POTENZE

$$(\sqrt{4}\sqrt{3})^x = 12^2$$

$$(\sqrt{12})^x = 12^2$$

$$\left((12)^{\frac{1}{2}}\right)^x = 12^2$$

$$12^{\frac{x}{2}} = 12^2 \xrightarrow[\text{INIETT.}]{} \frac{x}{2} = 2 \to x = 4$$

$$x = \log_{2\sqrt{3}}(144) = \frac{\log_{12}(144)}{\log_{12}(2\sqrt{3})} = \frac{\log_{12}(12^2)}{\log_{12}(12^{\frac{1}{2}})}$$

$$= \frac{2}{\frac{1}{2}} = 2 \cdot 2 = 4$$

Tracciare il grafico della curva $y = \log_{\frac{2}{3}}(x)$

$\log_{\frac{2}{3}}(1) = 0$

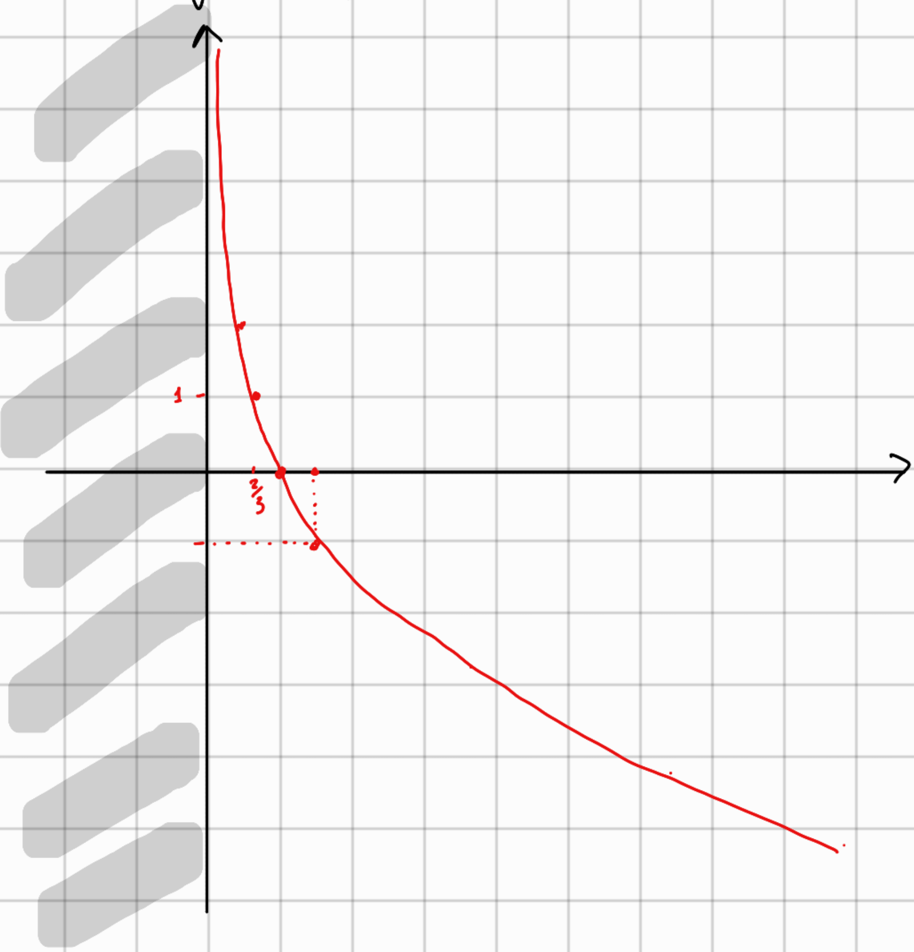
$\log_{\frac{2}{3}}\left(\frac{2}{3}\right) = 1$

$\log_{\frac{2}{3}}\left(\left(\frac{2}{3}\right)^2\right) = 2$

$\log_{\frac{2}{3}}\left(\left(\frac{2}{3}\right)^{-1}\right) = -1$

$\shortparallel$

$\log_{\frac{2}{3}}\left(\frac{3}{2}\right) = -1$



- $\dfrac{1}{|e^x - 1|} = 1$

$|\cdot| : \mathbb{R} \to [0, +\infty)$

$x \longmapsto \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$

C.E. $e^x - 1 \neq 0$

$\begin{cases} \dfrac{1}{e^x - 1} = 1 \\ e^x - 1 > 0 \end{cases} \vee \begin{cases} \dfrac{1}{-e^x + 1} = 1 \\ e^x - 1 < 0 \end{cases}$

$\downarrow \leftarrow$ SE VALGONO LE C.E.

$\begin{cases} 1 = e^x - 1 \\ e^x > 1 = e^0 \end{cases} \vee \begin{cases} 1 = -e^x + 1 \\ e^x < 1 \end{cases}$

$$\begin{cases} e^x = 2 \\ x > 0 \end{cases} \qquad \begin{cases} 0 = -e^x \\ x > 0 \end{cases} \quad e^x = 0$$

$$\downarrow$$

$$\begin{cases} x = \ln(2) \\ x > 0 \end{cases}$$

$$S_1 = \{\ln(2)\} \qquad S_2 = \emptyset$$

$$S = S_1 \cup S_2 = \{\ln(2)\}$$

$a > 0,\ a \neq 1$

- $\log_a(x^2+x-6) = \log_a(x-2) + \log_a(2x+1)$

$$\log_a(x^2+x-6) = \log_a[(x-2)(2x+1)]$$

$$x^2+x-6 = (x-2)(2x+1)$$

$$\cancel{(x-2)}(x+3) = \cancel{(x-2)}(2x+1)$$

SI PUÒ FARE SOLO SE
$x-2 \neq 0$
$x \neq 2$

$$x+3 = 2x+1$$
$$x-2 = 0$$
$$x = 2 \notin (2, +\infty)$$

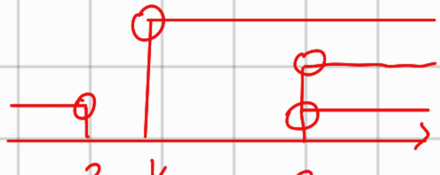
$$S = \emptyset$$

$\Delta = 1 + 24 = 25$

$x_{1,2} = \frac{-1 \pm 5}{2}$ ↗ $x_1 = -3$, ↘ $x_2 = 2$

C.E. $\begin{cases} x^2+x-6 > 0 \\ x-2 > 0 \\ 2x+1 > 0 \end{cases}$

$\begin{cases} x < -3 \vee x > 2 \\ x > 2 \\ x > -\frac{1}{2} \end{cases}$



$D: (2, +\infty)$

$$[\ln(x)]^2 = \ln^2 x$$

- $\ln^2 x + \ln x - 6 = 0$

C.E. $x > 0$

$$t^2 + t - 6 = 0$$

$t = \ln(x)$

$$\downarrow$$

$$\Delta = 1 + 24 = 25$$

$$t_{12} = \frac{-1 \pm 5}{2} \begin{array}{l} \nearrow t_1 = -3 \\ \searrow t_2 = 2 \end{array}$$

$$t = -3 \quad \vee \quad t = 2$$

$$\ln(x) = -3 \quad \vee \quad \ln(x) = 2$$

$$e^{\ln(x)} = e^{-3} \qquad e^{\ln x} = e^2$$

$$\downarrow \qquad \qquad \downarrow$$

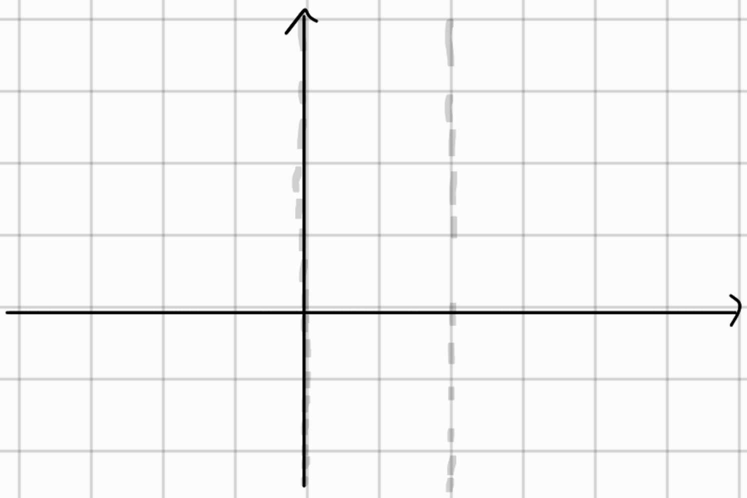
$$x = e^{-3} \quad \vee \quad x = e^2$$

$$x = \frac{1}{e^3}$$

$$S = \{e^{-3}, e^2\}$$

- TROVA IL DOMINIO DELLA FUNZIONE

$$f(x) = \frac{\ln|x|}{x-2}$$



$$\begin{cases} x - 2 \neq 0 \\ |x| > 0 \end{cases} \rightarrow \begin{cases} x \neq 2 \\ x \neq 0 \end{cases}$$

$$D = \mathbb{R} \setminus \{0, 2\}$$

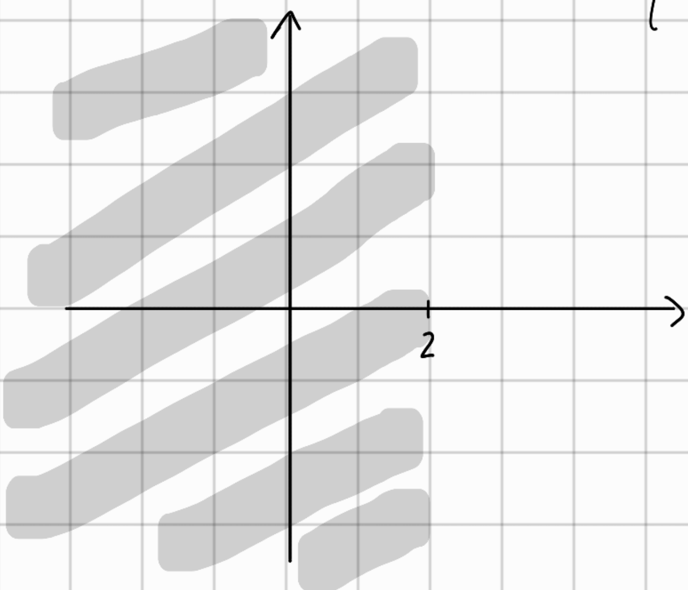
$$= (-\infty, 0) \cup (0, 2) \cup (2, +\infty)$$

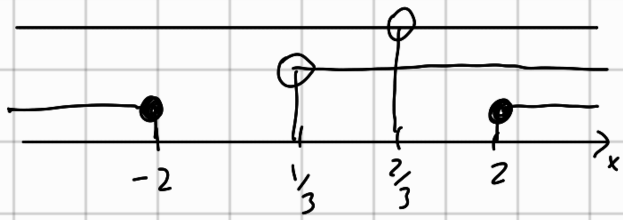
$$f(x) = \frac{\sqrt{x^2-4}}{\ln(3x-1)}$$

C.E $\begin{cases} \ln(3x-1) \neq 0 \\ 3x-1 > 0 \\ x^2-4 \geq 0 \end{cases}$ $\quad e^{\ln(3x-1)} \neq e^0$

$3x-1 \neq 1$

$$\begin{cases} 3x-1 \neq 1 \\ x > \frac{1}{3} \\ x \leq -2 \ \vee \ x \geq 2 \end{cases} \rightarrow \begin{cases} x \neq \frac{2}{3} \\ x > \frac{1}{3} \\ x \leq -2 \ \vee \ x \geq 2 \end{cases}$$





$x \geq 2$

$D = [2, +\infty)$

PROBLEMA

Il numero di batteri presenti in una coltura è 1000.
Sappiamo che $\boxed{\text{raddoppiano ogni due ore}}$

Quanti batteri ci sono dopo tre ore e mezza?

$N_0 = 1000$

$$N(t) = N_0 R^t$$

$t :=$ ore trascorse

$$\boxed{N(t+2) = 2N(t)} \longleftarrow$$

$R =$ LO RICAVO DA QUESTA INFORMAZIONE

$$\cancel{N_0} R^{t+2} = 2 \cancel{N_0} R^t$$

$$R^{t+2} = 2R^t \qquad R^t \neq 0$$

$$\frac{R^{t+2}}{R^t} = 2$$

$$R^{\cancel{t}+2-\cancel{t}} = 2$$

$$R^2 = 2$$

$$R = \pm\sqrt{2} \qquad -\sqrt{2} \text{ NON HA SENSO}$$

$$R = \sqrt{2}$$

$$N(t) = 1000 \cdot \sqrt{2}^{\,t} = 1000 \cdot 2^{t/2}$$

$$N(3.5) = 1000 \cdot 2^{\frac{3.5}{2}} = 1000 \cdot 2^{1.75} = 1000 \cdot (3.364) = 3364$$

$$\frac{e^{\sqrt{1-x}} - e^{2-2x}}{\left(\frac{1}{10}\right)^{x^2-3} - 10^{2x}} \leq 0$$

DEN

C.E. $\begin{cases} 1-x \geq 0 \\ \text{DEN} \neq 0 \end{cases}$ $\qquad \underline{x \leq 1}$

LA INSERISCO DOPO

$\cdot \; e^{\sqrt{1-x}} - e^{2-2x} \geq 0$

$e^{\sqrt{1-x}} \geq e^{2-2x}$

e^x MONOTONA CRESCENTE

$$\boxed{\sqrt{1-x} \geq 2-2x}$$

$\left(\frac{1}{10}\right)^{x^2-3} - 10^{2x} > 0$

$\left(\frac{1}{10}\right)^{x^2-3} - \left(\frac{1}{10}\right)^{-2x} > 0$

$\left(\frac{1}{10}\right)^{x^2-3} > \left(\frac{1}{10}\right)^{-2x}$

$\frac{1}{10} < 1$

$\frac{1}{10}^x$ è MONOTONA DECRESCENTE

$x^2 - 3 < -2x$

$x^2 + 2x - 3 < 0$

$$\begin{cases} 1-x \geq 0 \\ 2-2x \geq 0 \\ 1-x \geq (2-2x)^2 \end{cases}$$

$$\begin{cases} x \leq 1 \\ x \leq 1 \\ \frac{3}{4} \leq x \leq 1 \end{cases}$$

$$1-x \geq 4-8x+4x^2$$

$$4x^2-7x+3 \leq 0$$

$$\Delta = 49-48 = 1$$

$$x_{12} = \frac{7 \pm 1}{8} \quad x_1 = \frac{3}{4} \quad x_2 = 1$$

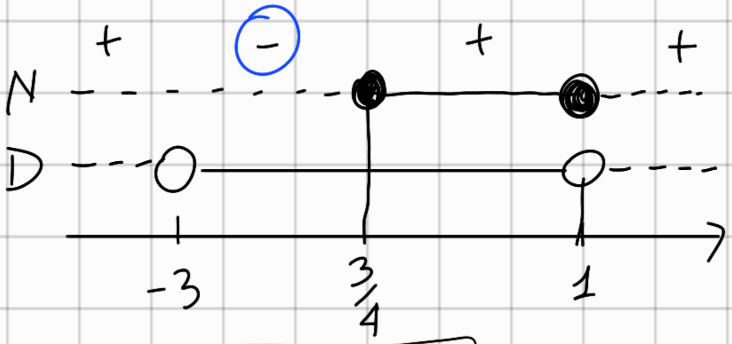
$$\frac{3}{4} \leq x \leq 1$$

$$\boxed{\frac{3}{4} \leq x \leq 1}$$

$$\Delta = 4+12 = 16$$

$$x_{12} = \frac{-2 \pm 4}{2} \quad x_1 = -3 \quad x_2 = 1$$

$$\boxed{-3 < x < 1}$$



$$\boxed{S = \left(-3, \frac{3}{4}\right]}$$

SODDISFA LE C.E. $x \leq 1$? SI