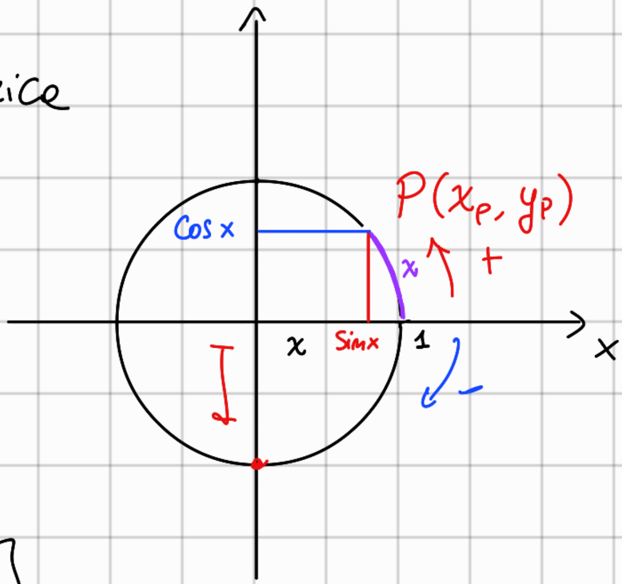
Lezione 11 25/10/24

FUNZIONI TRIGONOMETRICHE

C circonferenza goniometrica
centro O e raggio 1.



$$C = \{ (x_P, y_P) \in \mathbb{R}^2,\ x_P^2 + y_P^2 = 1 \}$$

$$\sin : \mathbb{R} \longrightarrow [-1, 1]$$
$$x \longmapsto \sin(x)$$

è la distanza del punto P dove mi trovo dopo aver percorso un angolo x dall'asse delle ascisse.

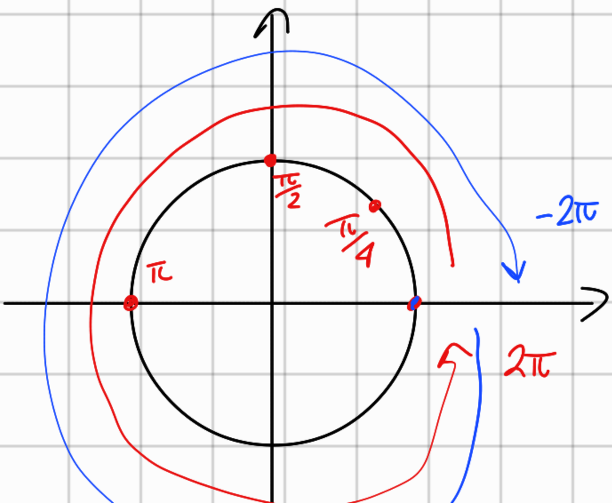
$$\cos : \mathbb{R} \longrightarrow [-1, 1]$$
$$x \longmapsto \cos x$$

è la distanza del punto P dove mi trovo dopo aver percorso un angolo x dall'asse delle ordinate.

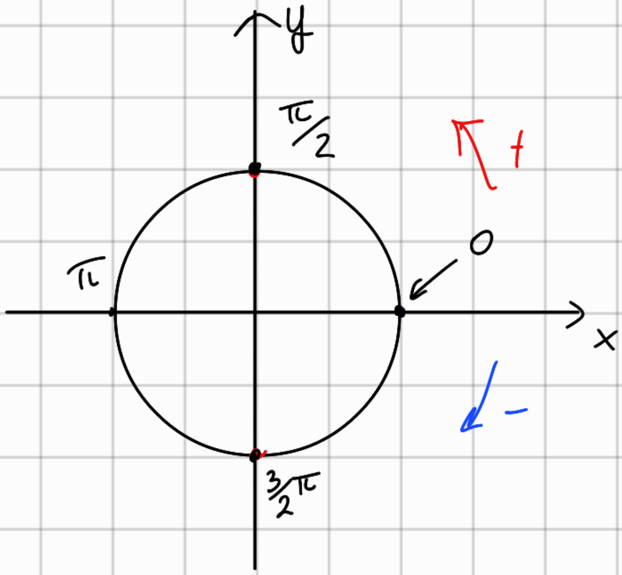
---

lunghezza Circonferenza $2\pi r = 2\pi$ (raggio)

$\frac{\pi}{2}$



DISEGNARE I SEGUENTI ANGOLI $-\pi, \frac{3}{2}\pi, -\frac{2}{3}\pi, \frac{\pi}{4}, \frac{7}{4}\pi, -\frac{\pi}{6}, \frac{5}{6}\pi, -\frac{\pi}{4}$

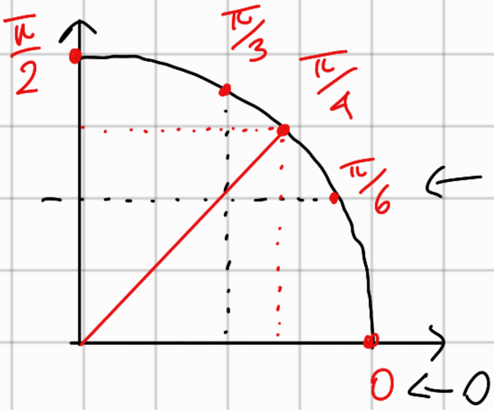


	x	$\sin x$	$\cos x$
	0	0	1
$-\frac{3}{2}\pi$	$\pi/2$	1	0
$-\pi$	π	0	-1
$-\frac{\pi}{2}$	$\frac{3}{2}\pi$	-1	0

EQUAZIONE FONDAMENTALE DELLA TRIGONOMETRIA

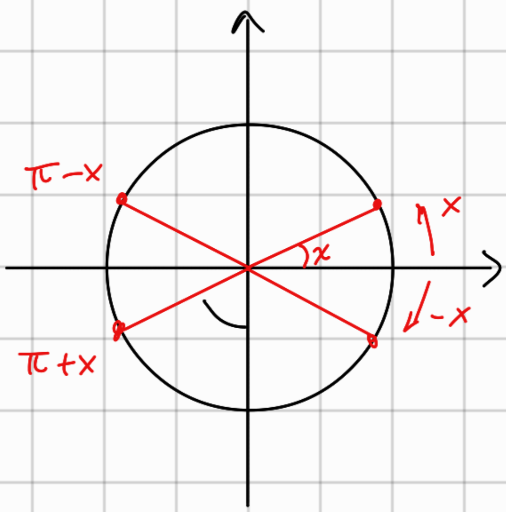
$\forall x \in \mathbb{R}$ $\quad \boxed{\sin^2 x + \cos^2 x = 1}$

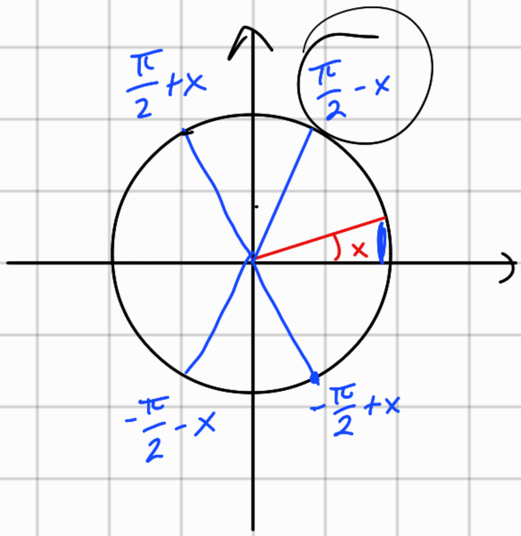
ANGOLI NOTEVOLI



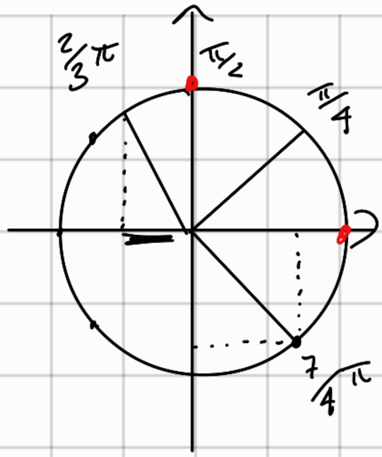
x	$\sin x$	$\cos x$
0	0 $\frac{\sqrt{0}}{2}$	1
$\pi/6$	$1/2$ $\sqrt{1}/2$	$\sqrt{3}/2$
$\pi/4$	$\sqrt{2}/2$ $\sqrt{2}/2$	$\sqrt{2}/2$
$\pi/3$	$\sqrt{3}/2$ $\sqrt{3}/2$	$1/2$
$\pi/2$	1 $\sqrt{4}/2$	0

FORMULE DEGLI ARCHI ASSOCIATI





x	$\sin x$	$\cos x$
$\pi - x$	$\sin x$	$-\cos x$
$-x$	$-\sin x$	$\cos x$
$\pi + x$	$-\sin x$	$-\cos x$
$\frac{\pi}{2} - x$	$\cos x$	$\sin x$
$\frac{\pi}{2} + x$	$\cos x$	$-\sin x$
$-\frac{\pi}{2} - x$	$-\cos x$	$-\sin x$
$-\frac{\pi}{2} + x$	$-\cos x$	$\sin x$



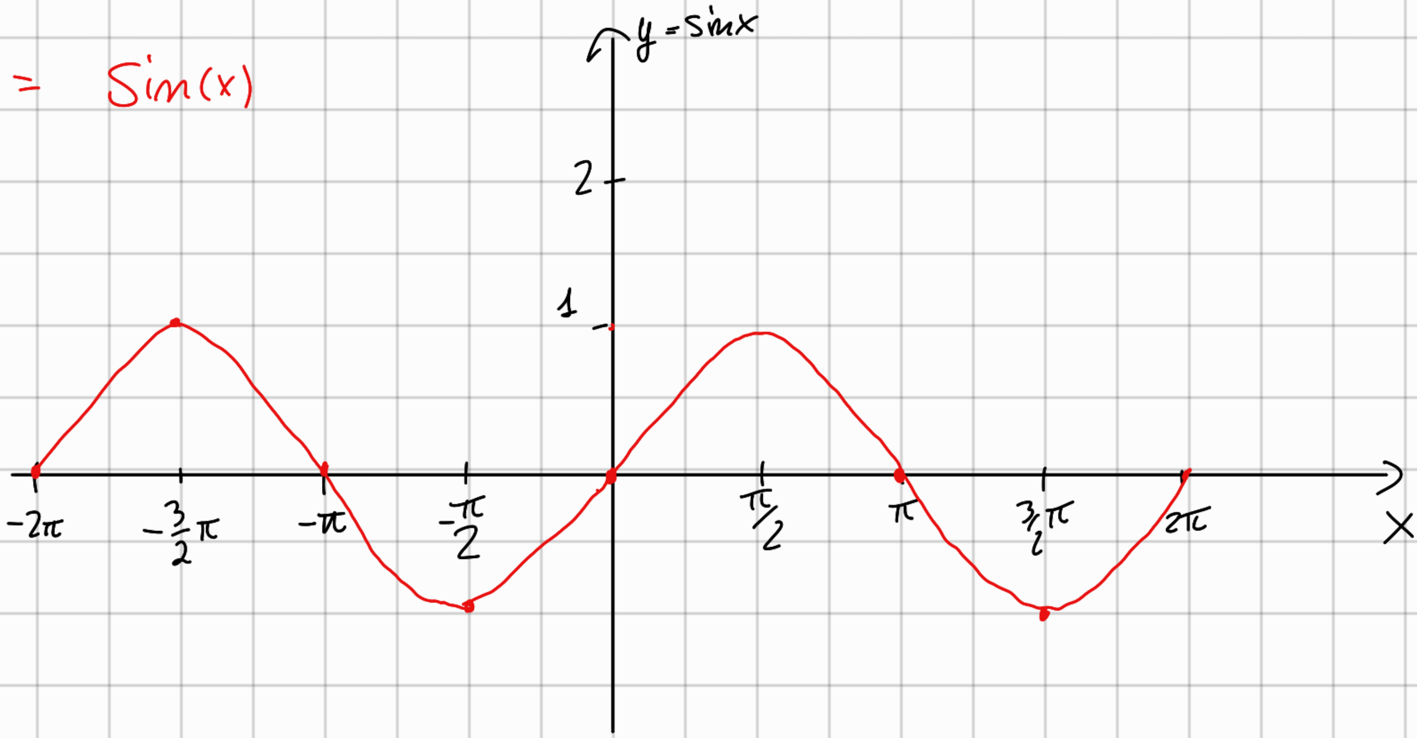
$$\cos\left(\frac{2}{3}\pi\right) = -\frac{1}{2}$$

$$\sin\left(\frac{2}{3}\pi\right) = \frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{2}{3}\pi\right) = \cos\left(\pi - \frac{\pi}{3}\right) = -\cos\left(\frac{\pi}{3}\right) = -\frac{1}{2}$$

PER CASA TROVARE IL SENO E IL COSENO DEGLI ANGOLI SCRITTI PRIMA

$y = \sin(x)$

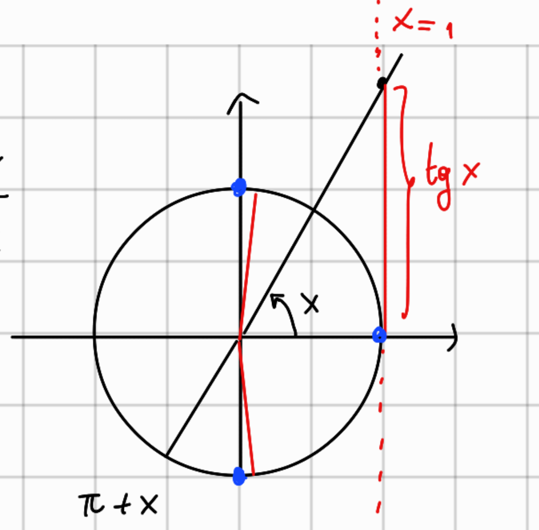


$$f: \mathbb{R} \setminus \left\{\frac{\pi}{2}+k\pi,\ k\in\mathbb{Z}\right\} \to \mathbb{R}$$

$$x \longmapsto \operatorname{tg} x = \frac{\sin x}{\cos x}$$

$$\left\{\dots, -\frac{3}{2}\pi, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3}{2}\pi, \frac{5}{2}\pi, \dots\right\}$$

$k=0$ $k=1$



SI PUÒ DIMOSTRARE USANDO

$$\frac{\sin(\pi+x)}{\cos(\pi+x)}$$

$$\operatorname{tg}(\pi+x) = \operatorname{tg}(x)$$

SIGNIFICA CHE

La funzione tangente è periodica con periodo π

Regole ④ delle C.E.

$$\operatorname{tg}(f(x)) \Rightarrow \quad \text{C.E.} \quad f(x) \neq \frac{\pi}{2}+k\pi \quad \forall k\in\mathbb{Z}$$

ESEMPIO

$$f(x) = \operatorname{tg}(x^2-1) \qquad x\in[-\pi,\pi]$$

$k\in\mathbb{Z}$

C.E. $x^2-1 \neq \frac{\pi}{2}+k\pi$

CERCO SOLO LE SOLUZIONI CHE APPARTENGONO ALL'INTERVALLO INDICATO

$$x^2 \neq 1+\frac{\pi}{2}+k\pi$$

$$x \neq \pm\sqrt{1+\frac{\pi}{2}+k\pi}$$

$\boxed{x \neq \pm\sqrt{1-\frac{\pi}{2}}}$, $x \neq \pm\sqrt{1+\frac{\pi}{2}}$, $x \neq \pm\sqrt{1+\frac{3}{2}\pi}$, $x \neq \pm\sqrt{1+\frac{5}{2}\pi}$

$k=-1 \qquad k=0 \qquad k=1 \qquad k=2$

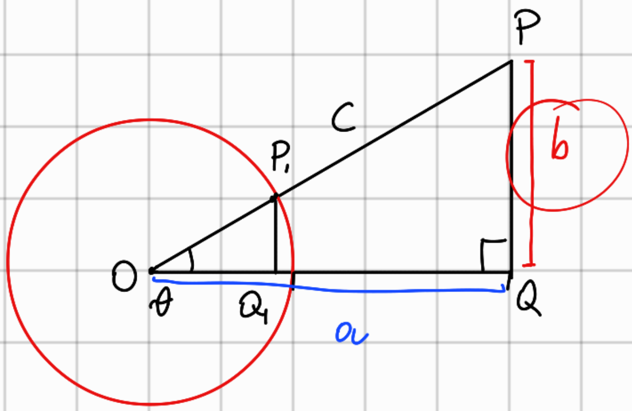
POICHÉ $1-\frac{\pi}{2}<0$ QUESTA CONDIZIONE NON HA SENSO

PER $k=3$ $\quad x \neq \pm\sqrt{1+\frac{\pi}{2}+3\pi} \rightarrow x \neq \pm\sqrt{1+\frac{\pi+6\pi}{2}} \rightarrow x \neq \sqrt{1+\frac{7}{2}\pi} \notin [-\pi,\pi]$

NON FA PARTE DELL'INTERVALLO CONSIDERATO

$$D = [-\pi,\pi] \setminus \left\{ \pm\sqrt{1+\frac{\pi}{2}}, \pm\sqrt{1+\frac{3}{2}\pi}, \pm\sqrt{1+\frac{5}{2}\pi} \right\}$$

FORMULE TRIGONOMETRICHE PER TRIANGOLI RETTANGOLI



PER SIMILITUDINE

$$OP : OP' = OQ : OQ'$$

$$\left.\begin{array}{l} P_1Q_1 = \sin\vartheta \\ OQ_1 = \cos\vartheta \\ OP_1 = 1 \end{array}\right\} \longrightarrow c : 1 = a : \cos\vartheta$$

$$\begin{cases} a = c\cos\vartheta \\ b = c\sin\vartheta \end{cases}$$ ANALOGAMENTE

DA QUESTE DUE SI RICAVA

$$\frac{b}{a} = \frac{\sin\vartheta}{\cos\vartheta} = tg\,\vartheta$$

la lunghezza di un cateto è uguale al prodotto dell'ipotenusa per

- il seno dell'angolo opposto

(OPPURE) • il coseno dell'angolo adiacente