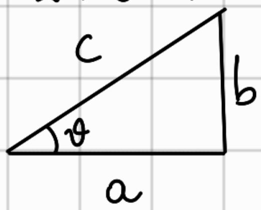
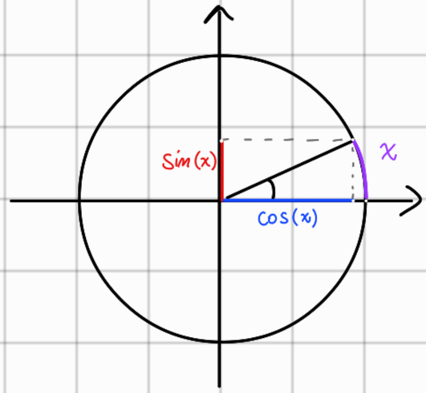
Lezione 12 28/10/24

RICORDO LA SCORSA LEZIONE





$$\begin{cases} a = c\cos\vartheta \\ b = c\sin\vartheta \end{cases}$$

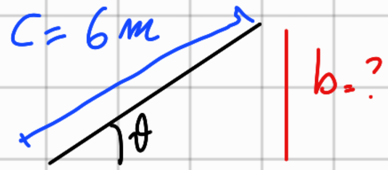
$$\downarrow$$

$$\frac{b}{a} = \operatorname{tg}\vartheta$$

ESEMPIO

Strada in salita

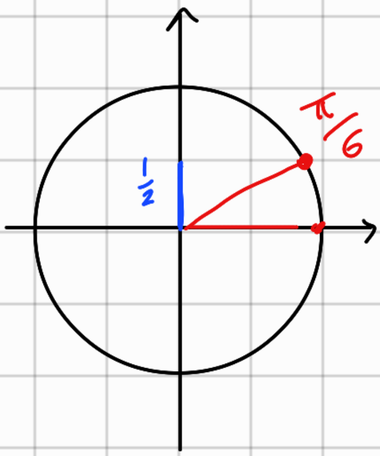
Sappiamo quanto è ripida la salita $\vartheta = \frac{\pi}{6}$ (30°)



Dopo aver percorso 6 metri (di rampa)

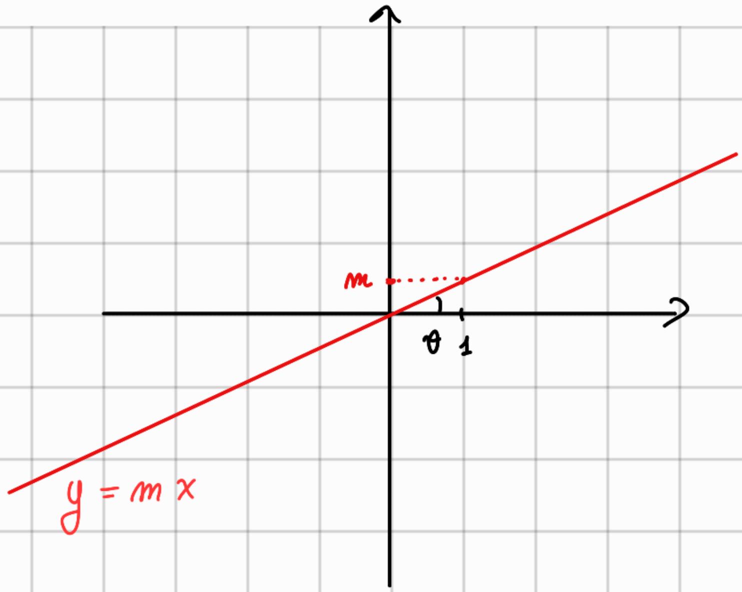
Quanto sarò salito?

$$b = c \cdot \sin\vartheta = 6m \cdot \frac{1}{2} = 3\ m$$

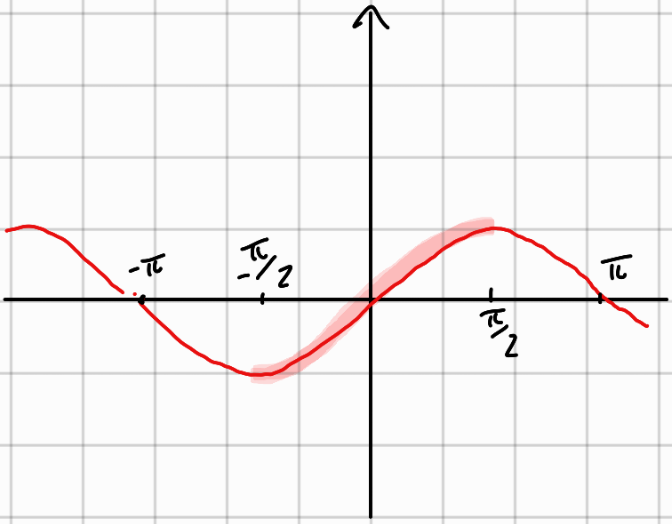


$m = tg\,\vartheta$

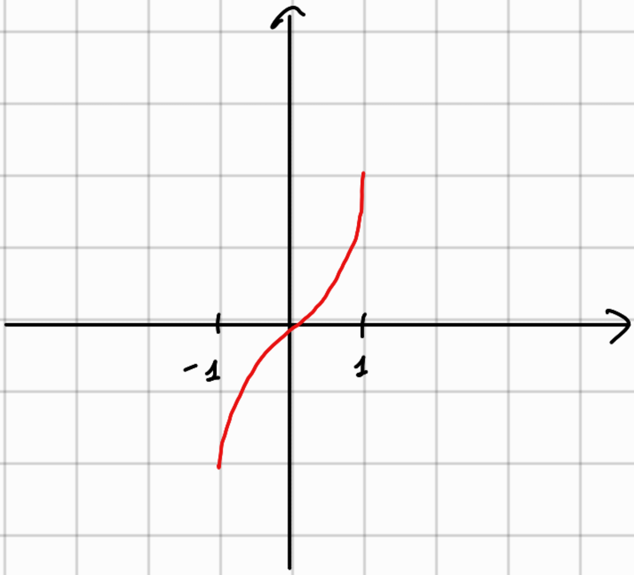
$\left(arctg(m) = \vartheta \right)$



FUNZIONI TRIGONOMETRICHE INVERSE



INVERTO

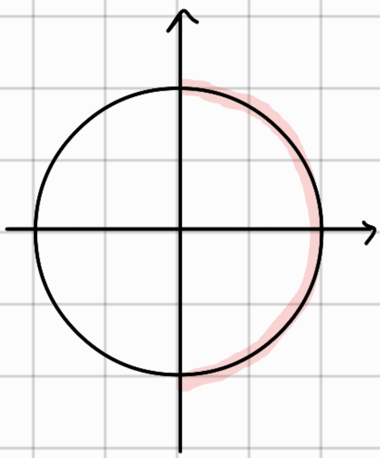


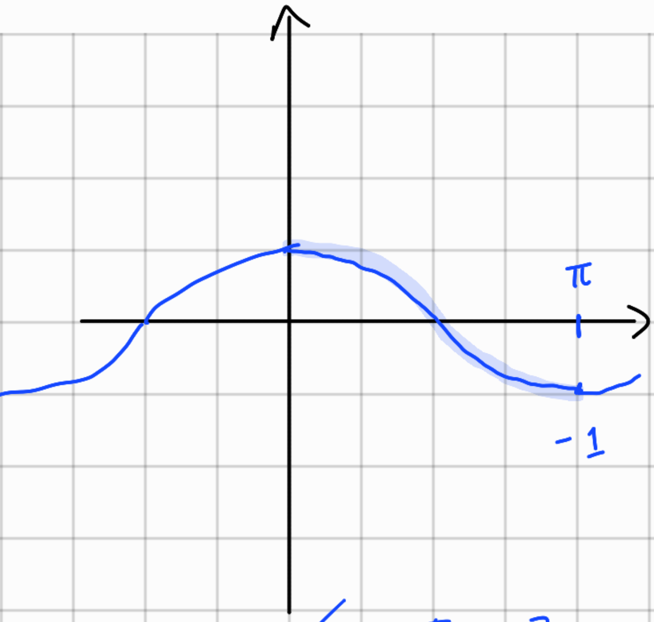
$\sin x : \cancel{\mathbb{R}} \to [-1, 1]$

$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to [-1, 1]$

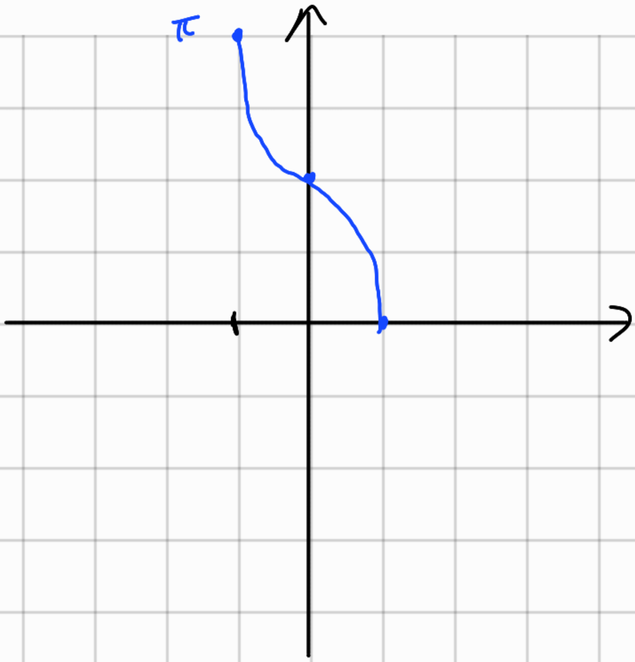
$x \longmapsto \sin x$

$\arcsin x : [-1, 1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

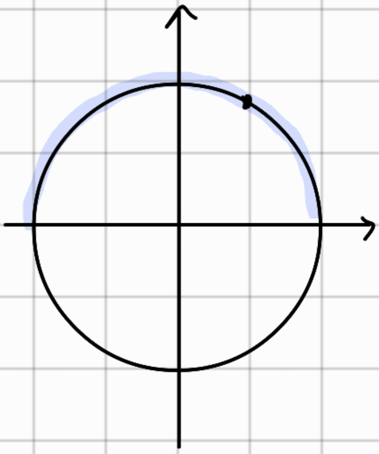




$\cos x$ $\cancel{\mathbb{R}}$ $[0,\pi] \longrightarrow [-1,1]$



$\arccos x : [-1,1] \longrightarrow [0,\pi]$

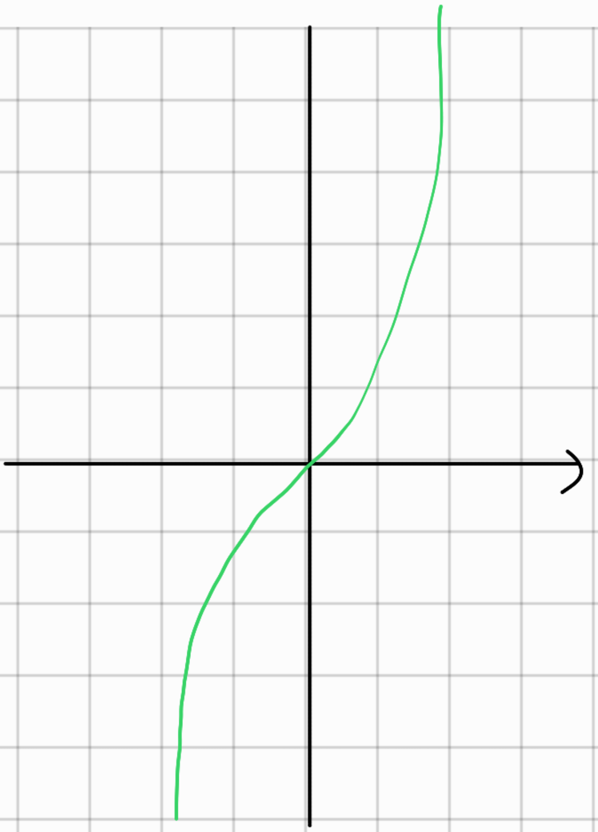


Regole (5) delle C.E.: se la funzione che studio contiene

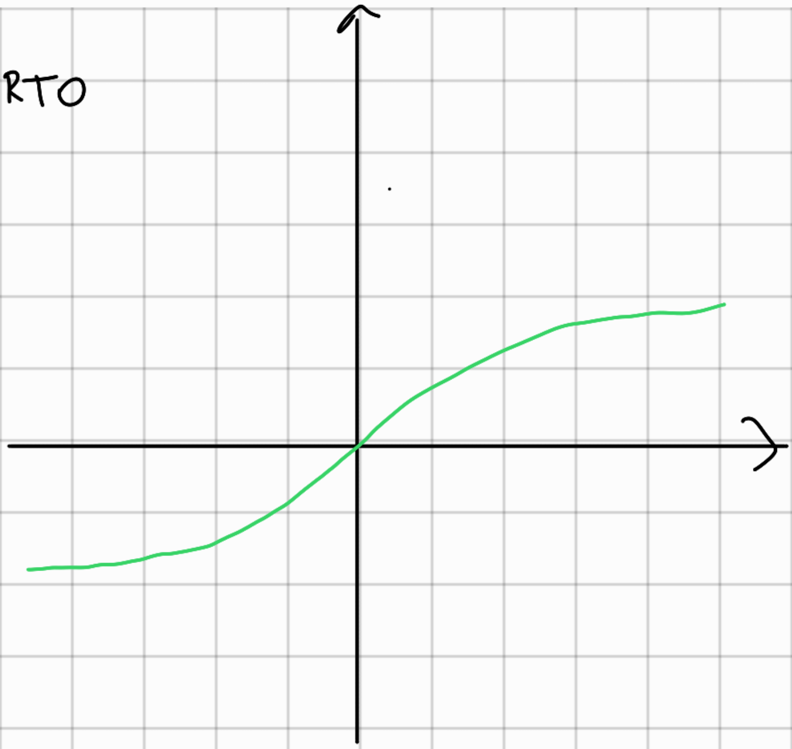
$\arcsin(f(x))$
$\arccos(f(x))$

$$\Downarrow$$

C.E. $\quad -1 \leq f(x) \leq 1$



INVERTO $\curvearrowright$



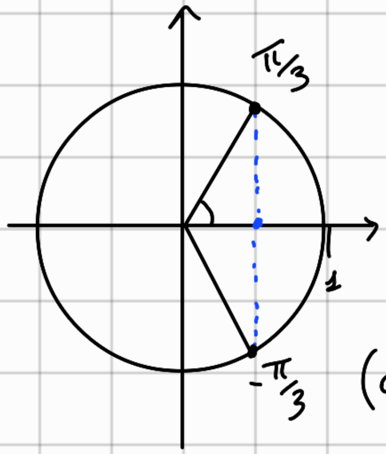
$tg : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to \mathbb{R}$

$arctg : \mathbb{R} \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$x \longmapsto arctg(x)$

$\cos x = \frac{1}{2} \qquad x \in \mathbb{R}$

$k \in \mathbb{Z}$



(oppure $\frac{5}{3}\pi$)

$x = \frac{\pi}{3} + 2k\pi \quad \vee \quad x = -\frac{\pi}{3} + 2k\pi$

$$S = \left\{ \frac{\pi}{3} + 2k\pi : k \in \mathbb{Z} \right\} \cup \left\{ -\frac{\pi}{3} + 2k\pi : k \in \mathbb{Z} \right\}$$

$\downarrow$

$$\left\{ \dots, -\frac{5}{3}\pi, \frac{\pi}{3}, \frac{7}{3}\pi, \dots \right\} \cup \left\{ \dots, -\frac{7}{3}\pi, -\frac{\pi}{3}, \frac{5}{3}\pi, \dots \right\}$$

$k=-1 \quad k=0 \quad k=1$

$\frac{\pi}{3} + 2\pi$

$\frac{1+6}{3}\pi = \frac{7}{3}\pi$

Supp $\quad x \in [-\pi, \pi]$

$\downarrow$

$$S = \left\{-\frac{\pi}{3}, \frac{\pi}{3}\right\} = \left\{\pm\frac{\pi}{3}\right\}$$

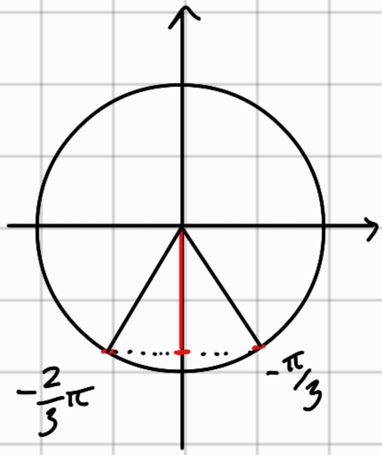
$$\frac{\pi}{3} - 2\pi = \frac{\pi - 6\pi}{3} = -\frac{5}{3}\pi$$

$$-\frac{\pi}{3} + 2\pi = \frac{5}{3}\pi$$

$$\sin x = -\frac{\sqrt{3}}{2} \qquad k \in \mathbb{Z}$$

$\downarrow$

$$x = -\frac{2}{3}\pi + 2k\pi \ \vee \ x = -\frac{\pi}{3} + 2k\pi$$



$\downarrow$

$\left(\text{oppure } \frac{4}{3}\pi\right)$

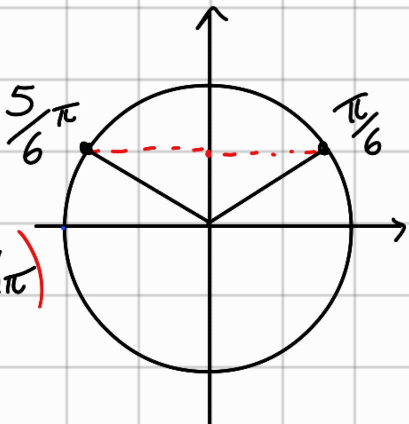
TROVARE IL DOMINIO DELLE SEGUENTI FUNZIONI

$$f(x) = \frac{5}{2\sin x - 1} \qquad x \in [-\pi, \pi]$$

$$2\sin x - 1 \neq 0$$

$$2\sin x \neq 1$$

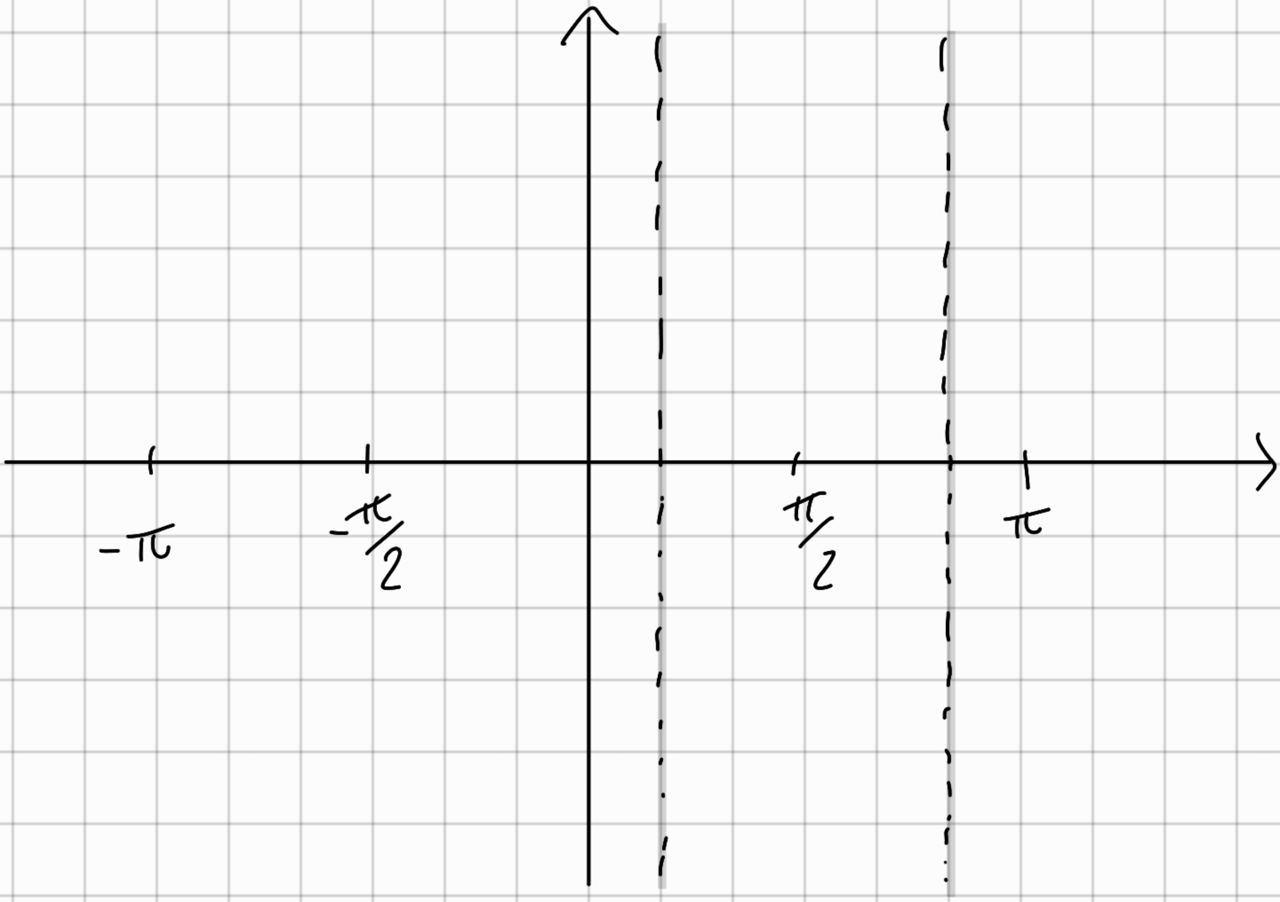
$$\sin x \neq \frac{1}{2}$$



$$x \neq \frac{5}{6}\pi \,(+2k\pi) \ \wedge \ x \neq \frac{\pi}{6}\,(+2k\pi)$$

$\uparrow$

$D = \left[-\pi, \frac{\pi}{6}\right) \cup \left(\frac{\pi}{6}, \frac{5}{6}\pi\right) \cup \left(\frac{5}{6}\pi, \pi\right]$



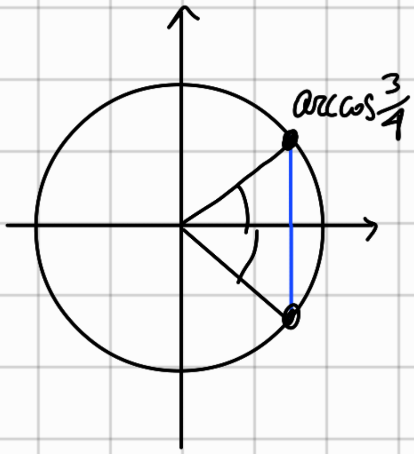
- $f(x) = \dfrac{3x+3}{\cos x - 2}$

$\cos x - 2 \neq 0$

$\cos x \neq 2$

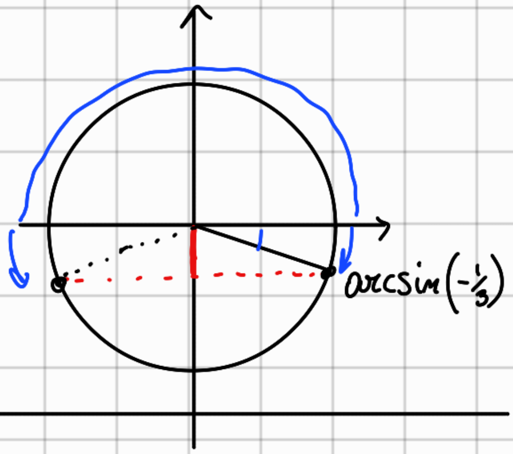
$D = \mathbb{R}$

- $\cos x = \frac{3}{4}$



$x = \arccos \frac{3}{4} + 2k\pi \ \vee \ x = -\arccos \frac{3}{4} + 2k\pi$

- $\sin x = -\frac{1}{3}$



$x = \arcsin -\frac{1}{3} + 2k\pi \ \vee \ \underline{\pi - \arcsin\left(-\frac{1}{3}\right)} + 2k\pi$

SE IN UN ESERCIZIO TROVO:

C.E. $\quad \sin x \neq -\frac{1}{3} \qquad x \in [-\pi, \pi]$

$\downarrow$

$\vartheta = \arcsin\left(-\frac{1}{3}\right) \simeq -0{,}339$

$x \neq \vartheta + 2k\pi \ \wedge \ x \neq \pi - \vartheta + 2k\pi = (2k+1)\pi - \vartheta$

$\{\ldots, -3\pi - \vartheta, -\pi - \vartheta, \pi - \vartheta, 3\pi - \vartheta, \ldots\}$

$\in [-\pi, \pi]$

$D = [-\pi, -\pi + \vartheta) \cup (-\pi - \vartheta, \vartheta) \cup (\vartheta, \pi]$

- $\operatorname{tg} x = 4$

$\downarrow$

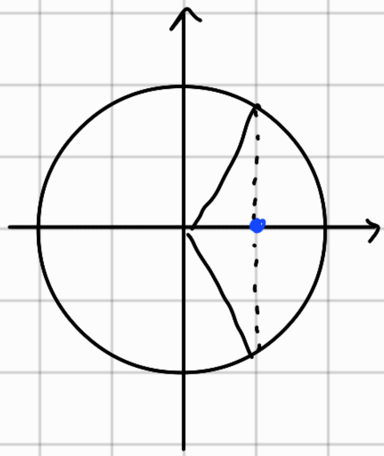
$x \neq \operatorname{arctg} 4 + k\pi \qquad k \in \mathbb{Z}$

ANGOLI NOTI

$0, \ \frac{\sqrt{3}}{3}, \ 1, \ \sqrt{3}, \ $ NON ESISTE

$0 \quad \frac{\pi}{6} \quad \frac{\pi}{4} \quad \frac{\pi}{3} \quad \frac{\pi}{2}$

- $\cos(3x) = \frac{1}{2}$ $\qquad$ $\underline{x \in [-\pi, \pi]}$



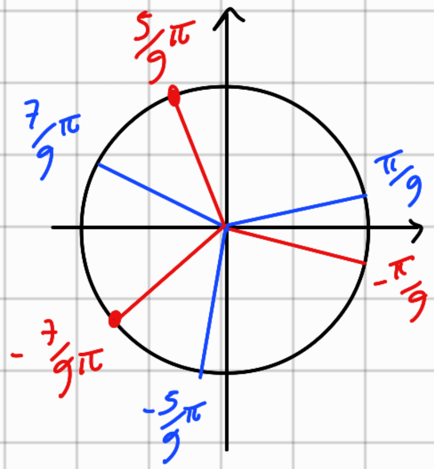
$$3x = \frac{\pi}{3} + 2k\pi \quad \vee \quad 3x = -\frac{\pi}{3} + 2k\pi$$

$$x = \frac{\pi}{9} + \frac{2}{3}k\pi \quad \vee \quad x = -\frac{\pi}{9} + \frac{2}{3}k\pi$$

$k=0 \rightarrow x = \frac{\pi}{9}$

$k = 1 \rightarrow x = \frac{\pi}{9} + \frac{2}{3}\pi = \frac{1+6}{9}\pi = \frac{7}{9}\pi$

$k = -1 \rightarrow x = \frac{\pi}{9} - \frac{2}{3}\pi = \frac{1-6}{9}\pi = -\frac{5}{9}\pi$



$k=0 \quad x = -\pi/9$

$k=1 \quad x = \frac{5}{9}\pi$

$k=-1 \quad x = -\frac{7}{9}\pi$

$\cancel{k=2 \quad x = -\frac{\pi}{9} + \frac{4}{3}\pi = \frac{-1+12}{9}\pi = \frac{11}{9}\pi \notin [-\pi, \pi]}$

• $f(x) = \dfrac{\cos x}{tg^2 x + 2tg x + 1}$

C.E. $\begin{cases} tg^2 x + 2tg x + 1 \neq 0 \\ x \neq \frac{\pi}{2} + k\pi \end{cases}$

$x \in [-\pi, \pi]$

$x \neq -\frac{\pi}{2} \wedge x \neq \frac{\pi}{2}$

$t = tg\, x$

$t^2 + 2t + 1 \neq 0$

$\Delta = 0$

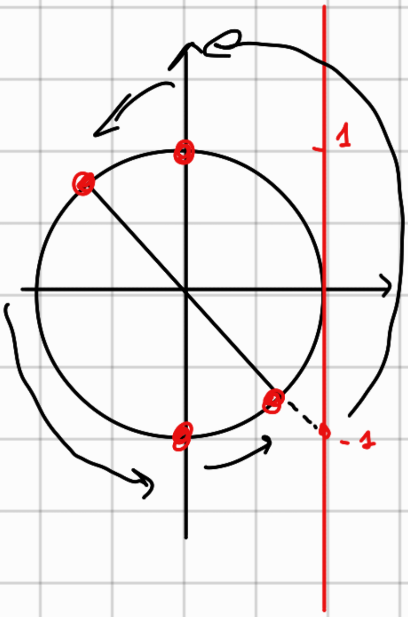
$(t+1)^2 \neq 0$

$t + 1 \neq 0$

$t \neq -1$

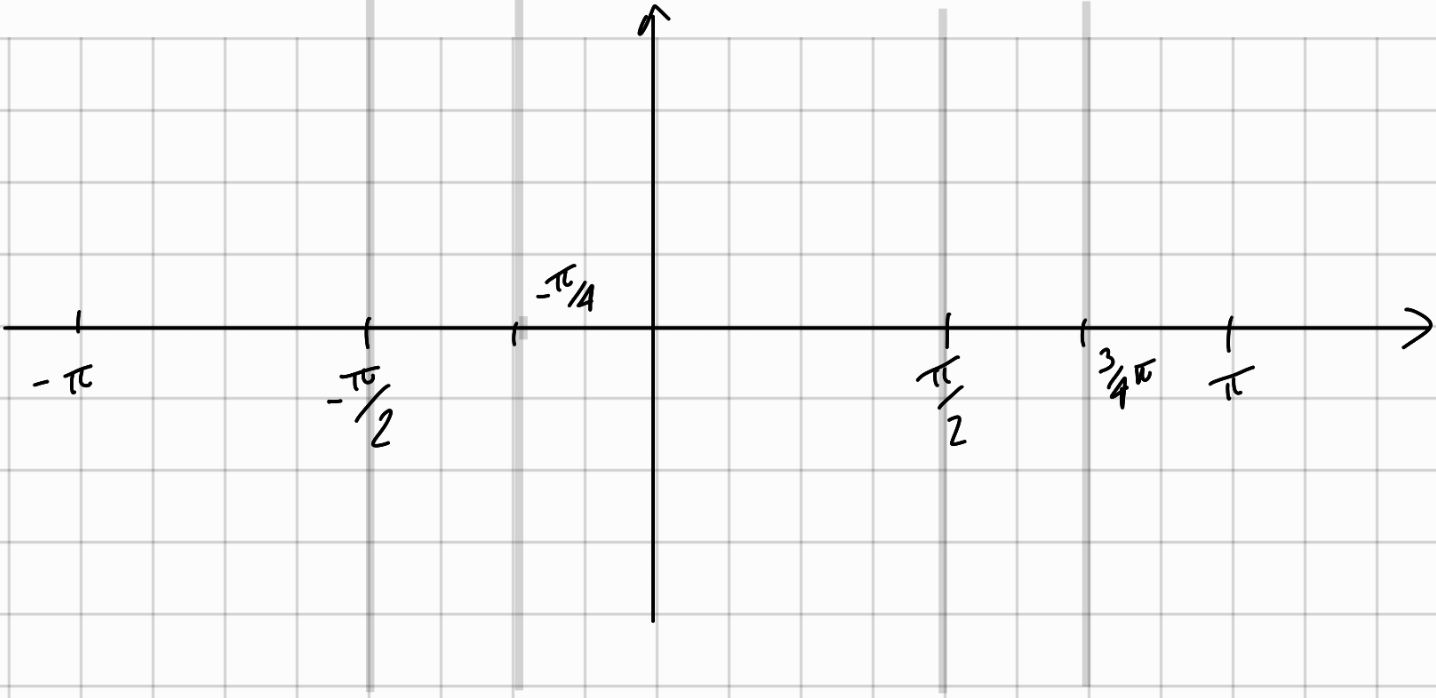
$tg\, x \neq -1$

$x \neq -\frac{\pi}{4} + k\pi$



$x \neq -\frac{\pi}{4} \wedge x \neq \frac{3}{4}\pi$

$D = \left[-\pi, -\frac{\pi}{2}\right) \cup \left(-\frac{\pi}{2}, -\frac{\pi}{4}\right) \cup \left(-\frac{\pi}{4}, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3}{4}\pi\right) \cup \left(\frac{3}{4}\pi, \pi\right]$



FORMULE DEL SENO e COSENO della SOMMA

$$\begin{bmatrix} \cos(\alpha \overset{(-)}{+} \beta) = \cos\alpha\cos\beta \overset{(+)}{-} \sin\alpha\sin\beta \\ \sin(\alpha \underset{(-)}{+} \beta) = \cos\alpha\sin\beta \underset{(-)}{+} \sin\alpha\cos\beta \end{bmatrix}$$

PER LA DIFFERENZA SI POSSONO RICAVARE DA $\begin{cases} \cos(-\alpha) = \cos\alpha \\ \sin(-\alpha) = -\sin\alpha \end{cases}$

$$\cos(\alpha + (-\beta)) = \cos\alpha\cos(-\beta) - \sin\alpha\sin(-\beta)$$

$$= \cos\alpha\cos\beta + \sin\alpha\sin\beta$$