Lezione 14 — 05/11/24

ESEMPIO CON NUMERI

$$f(x) = \frac{2x^2 - 6x}{x-3} \qquad D = \mathbb{R} - \{3\} = (-\infty, 3) \cup (3, +\infty)$$

NON POSSO FARE $f(3)$

MA VOGLIO SAPERE COSA SUCCEDE VICINO A 3

x	$f(x)$
2.9	5.8
2.99	5.98
2.999	5.998
3	?
3.001	6.002
3.01	6.02
3.1	6.2

Scelto un $\varepsilon > 0$, per esempio $\varepsilon = 0.002$

$$|f(x) - 6| < \varepsilon$$

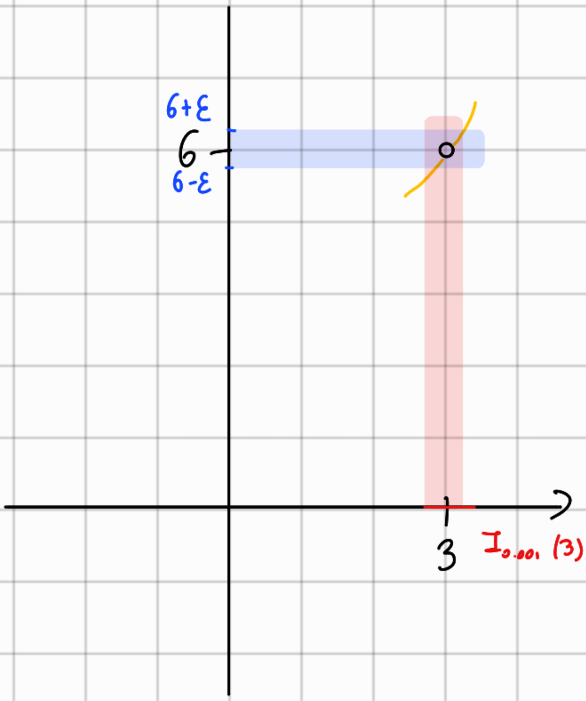
$$-\varepsilon < f(x) - 6 < \varepsilon$$

$$6 - \varepsilon < f(x) < 6 + \varepsilon$$

$$5.998 < f(x) < 6.002 \Rightarrow \underline{f(x) \in (5.998, 6.002)}$$

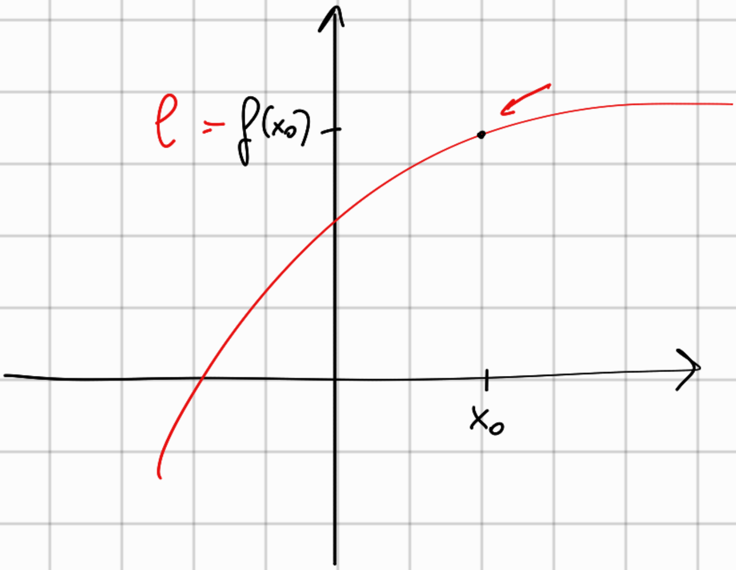
SE ESISTE $\delta > 0$ t.c. $\forall x \in I_\delta(3) \setminus \{3\}$

SCELGO $\delta = 0.001 \Rightarrow I_{0.001}(3) = (2.999, 3.001)$



DEFINIZIONE (DATO $x_0 \in D$, $f: D \to \mathbb{R}$ funzione)
Diremo che $f(x)$ è $\boxed{\text{CONTINUA}}$ in x_0 se

$$\lim_{x \to x_0} f(x) = f(x_0)$$



Se $\forall x_0 \in D$ f è CONTINUA IN $x_0 \Rightarrow f$ è CONTINUA

DEFINIZIONE

Data $f: D \to \mathbb{R}$ funzione, $x_0 \in \overline{D}$ diremo che IL LIMITE DESTRO (SINISTRO) per x che tende a x_0 (SI DENOTA $x \to x_0^{+(-)}$)

$$\lim_{x \to x_0^{+(-)}} f(x) = \ell$$

SE $\forall \varepsilon > 0 \quad \exists I_\delta^{+(-)}(x_0)$ t.c. $\forall x \in I_\delta^{+(-)}(x_0) \quad |f(x) - \ell| < \varepsilon$

$\boxed{\text{NELLA DEF. ORIGINALE } (x_0 - \delta, x_0 + \delta) \setminus \{x_0\}}$

$$I^+(x_0) = (x_0, x_0 + \delta)$$

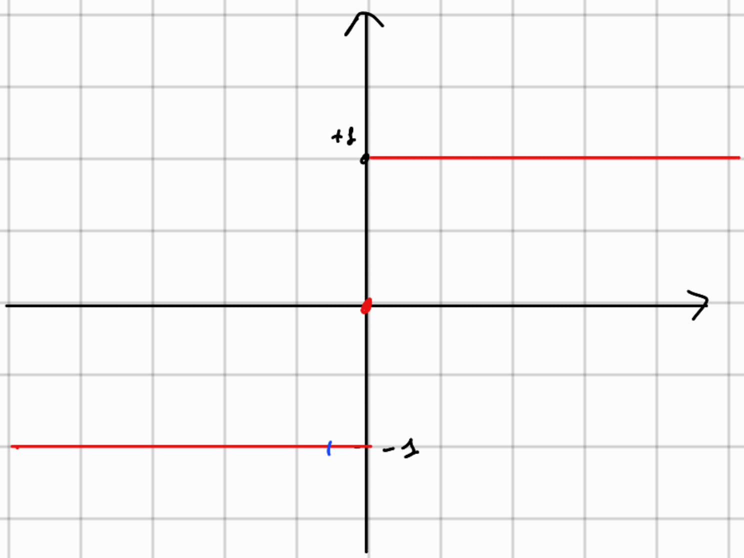
$$I^-(x_0) = (x_0 - \delta, x_0)$$

OSSERVAZIONE $\quad -5 + 0.001 = -4{,}999 \left(\simeq (-5)^+\right)$

ESEMPIO $\quad \operatorname{sgn}: \mathbb{R} \to \{-1, 0, 1\}$

$$x \longmapsto \operatorname{sgn}(x) = \begin{cases} -1 & \text{se } x < 0 \\ 0 & \text{se } x = 0 \\ 1 & \text{se } x > 0 \end{cases}$$

$$\nexists \lim_{x \to 0} \operatorname{sgn}(x)$$

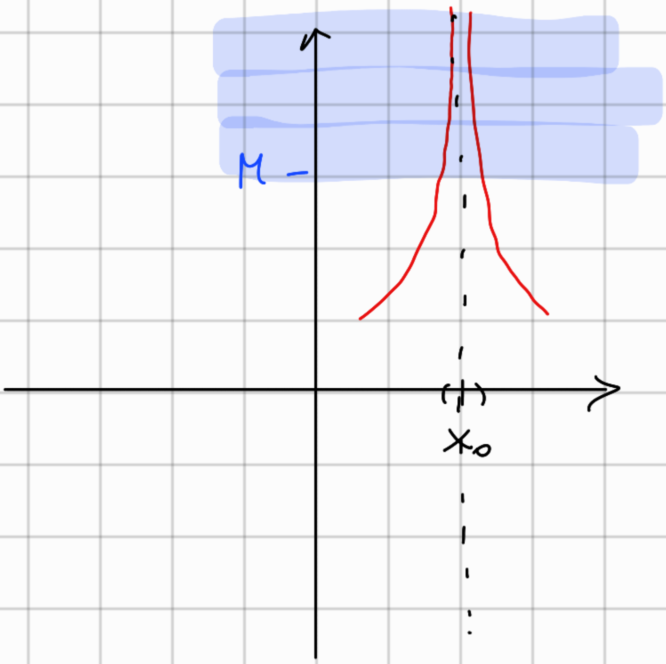


$$\lim_{x \to 0^-} \operatorname{sgn}(x) = -1$$

$$\lim_{x \to 0^+} \operatorname{sgn}(x) = +1$$

DEFINIZIONE [LIMITE $x \to x_0$ CON RISULTATO INFINITO]

DIREMO CHE $\lim_{x \to x_0} f(x) = +\infty$ $(-\infty)$



$\iff$

$\forall M > 0$ ($M < 0$) $\exists \delta > 0$ t.c.

$f(x) > M$ ($<$) $\forall x \in I_\delta(x_0) \setminus \{0\}$

SE a. $\lim_{x \to x_0^-} f(x) = \pm\infty \Rightarrow x = x_0$ è ASINTOTO VERTICALE SINISTRO

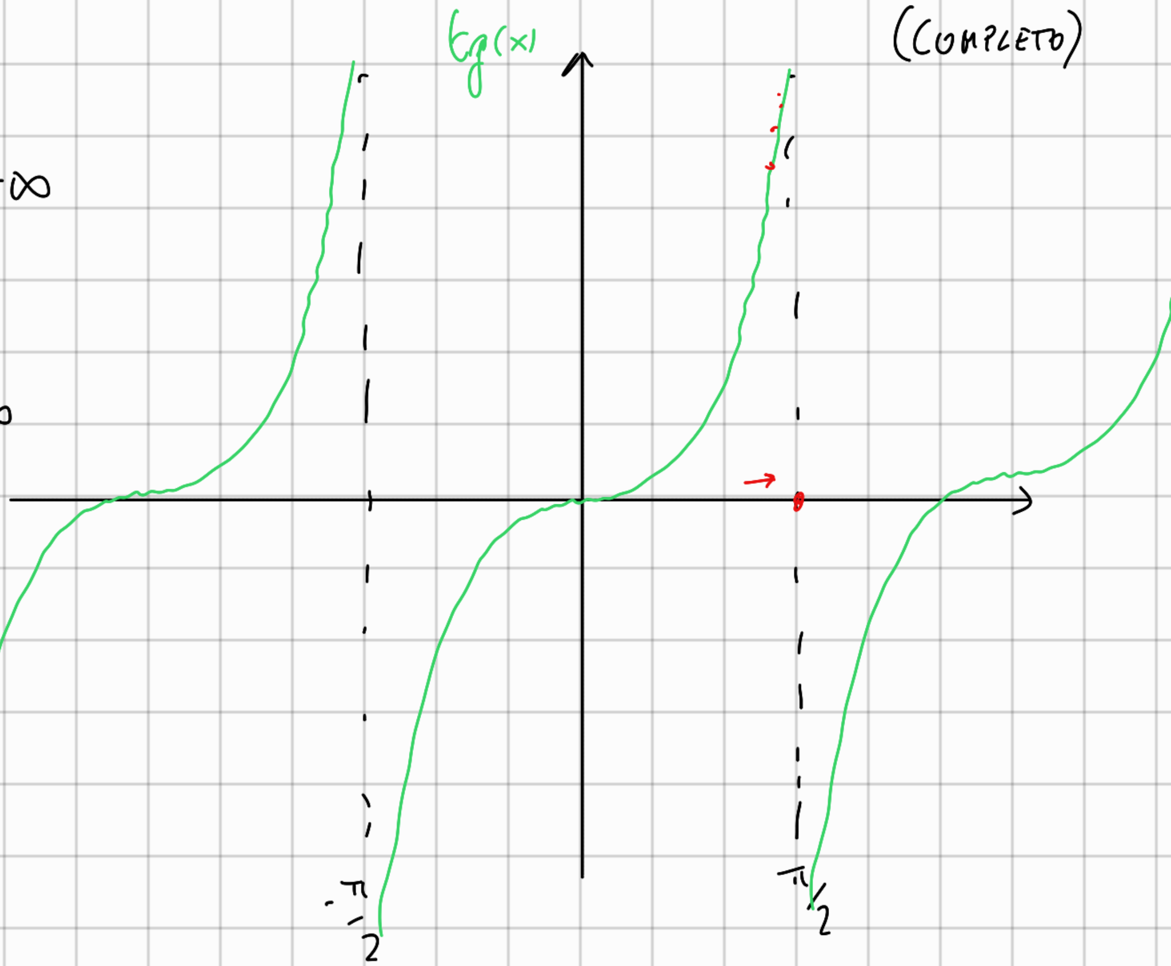
b. $\lim_{x \to x_0^+} f(x) = \pm\infty \Rightarrow x = x_0$ è ASINTOTO VERTICALE DESTRO

SE VALGONO a. e b. $x = x_0$ è ASINTOTO VERTICALE (COMPLETO)

$\lim_{x \to \frac{\pi}{2}^-} tg(x) = +\infty$

$\lim_{x \to \frac{\pi}{2}^+} tg(x) = -\infty$

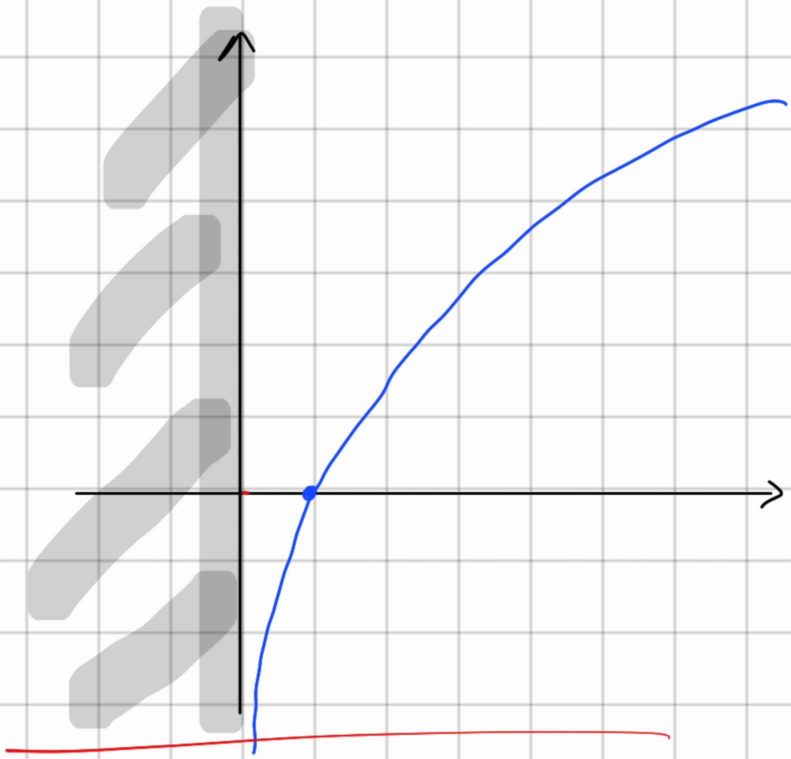
$x = \frac{\pi}{2}$ è A.V.



ESEMPIO

$f(x) = \ln(x) \qquad D = (0, +\infty)$

$$\boxed{\lim_{x \to 0^+} \ln(x) = -\infty}$$

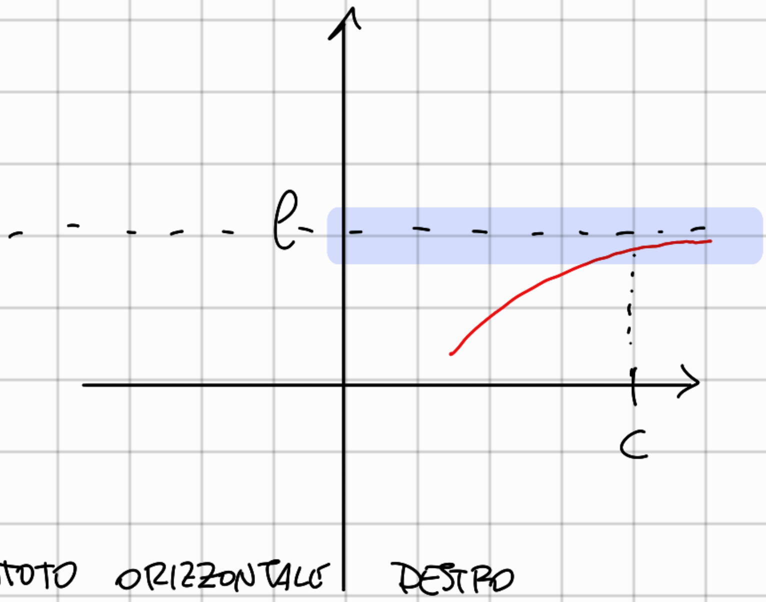


DEFINIZIONE $[x \to \pm\infty \quad f(x) \to \ell \in \mathbb{R}]$

DIREMO CHE $\lim_{x \to +\infty \,(-\infty)} f(x) = \ell$

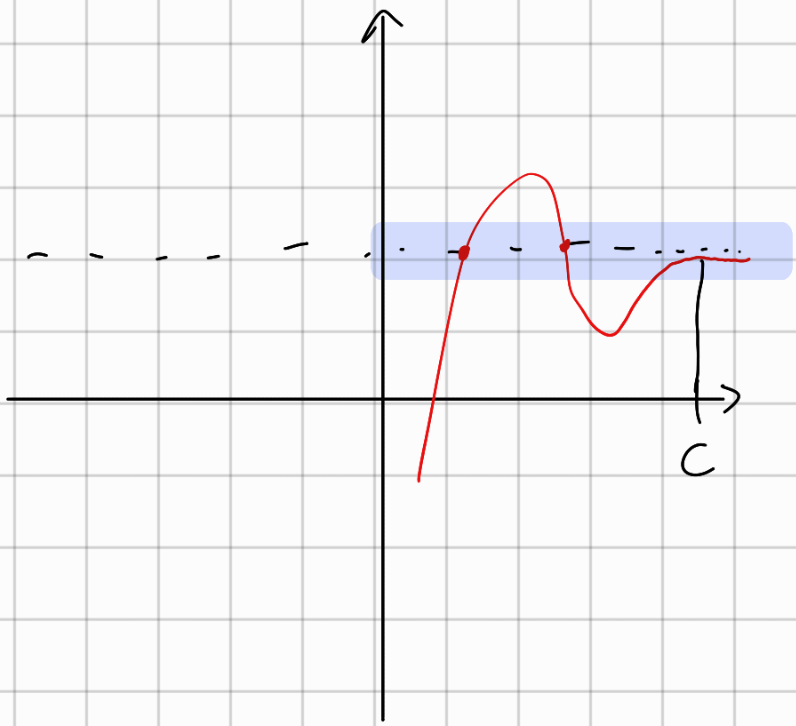
$\forall \varepsilon > 0 \quad \exists c > 0 \; (c < 0)$ t. c.

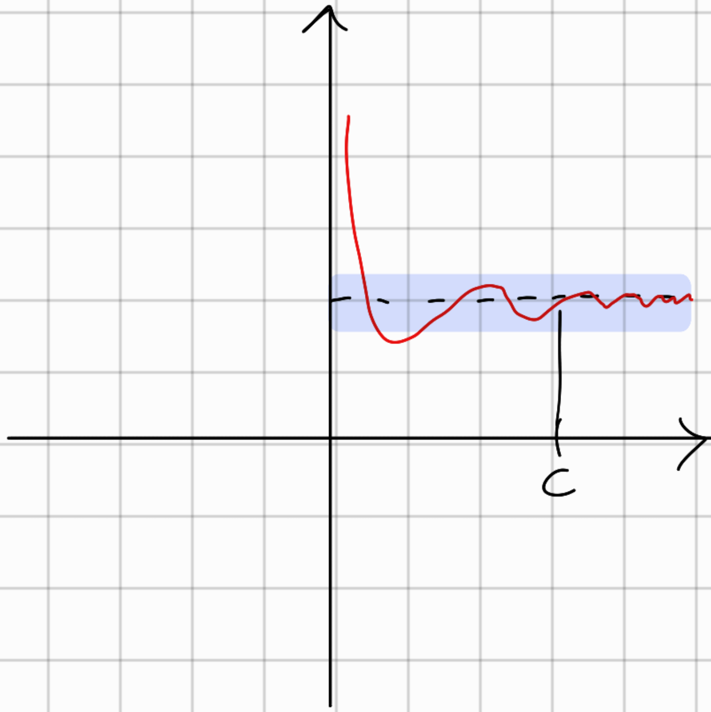
$|f(x) - \ell| < \varepsilon \quad \forall x > c \; (x < c)$



DIREMO CHE $y = \ell$ è ASINTOTO ORIZZONTALE DESTRO (SINISTRO)

AL CONTRARIO DELL'ASINTOTO VERTICALE LA FUNZIONE PUÒ INTERSECARE UN ASINTOTO ORIZZONTALE





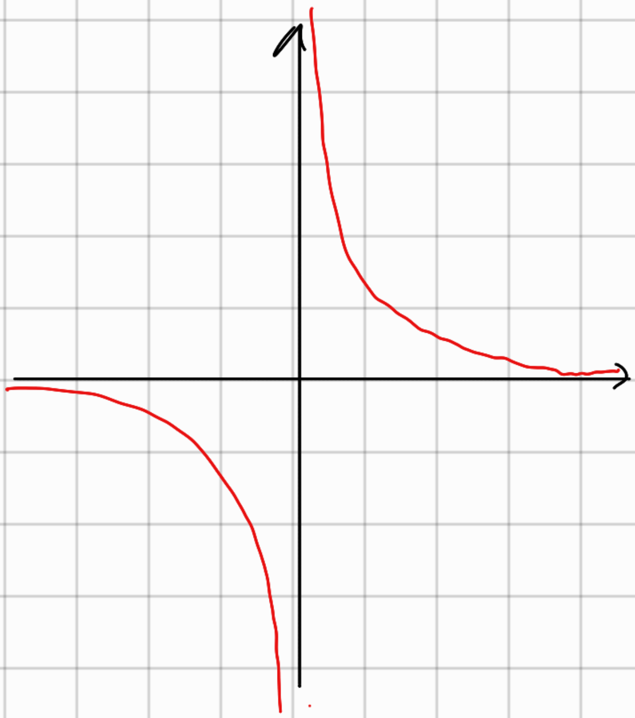
UN NUMERO FINITO DI VOLTE

O ANCHE INFINITE VOLTE

ESEMPIO $f(x) = \frac{1}{x}$

$$\left.\begin{aligned} \lim_{x \to 0^-} \frac{1}{x} &= -\infty \\ \lim_{x \to 0^+} \frac{1}{x} &= +\infty \end{aligned}\right] \quad x = 0 \ \text{A.V.}$$

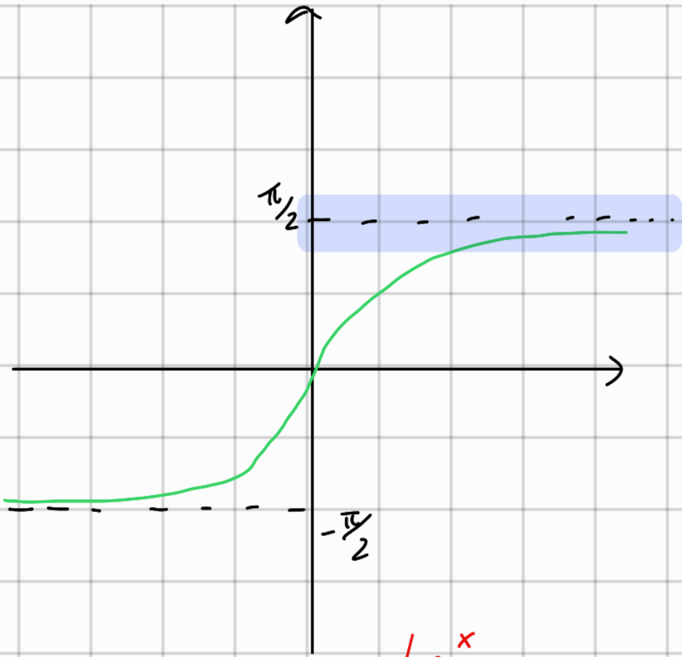
$$\left.\begin{aligned} \lim_{x \to +\infty} \frac{1}{x} &= 0 \\ \lim_{x \to -\infty} \frac{1}{x} &= 0 \end{aligned}\right] \quad y = 0 \ \text{A.OR.}$$



$f(x) = \operatorname{arctg}(x) \qquad \underset{(-\infty,+\infty)}{\mathbb{R}} \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\lim\limits_{x \to +\infty} \operatorname{arctg}(x) = \frac{\pi}{2} \to y = \frac{\pi}{2}$ A. OR. DX

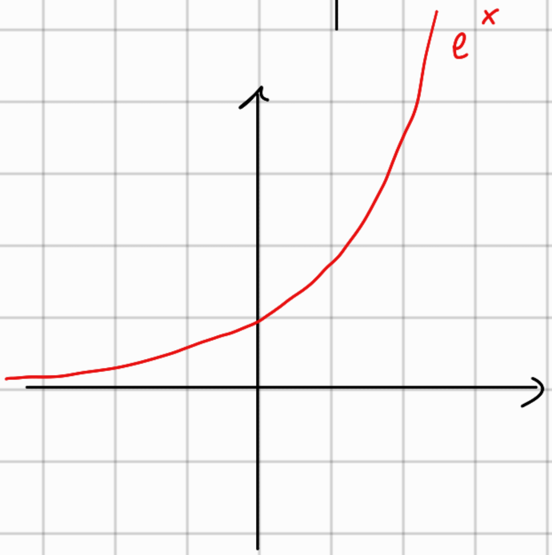
$\lim\limits_{x \to -\infty} \operatorname{arctg}(x) = -\frac{\pi}{2} \to y = -\frac{\pi}{2}$ A. OR. SX



ESEMPIO $\quad f(x) = e^x$

$\lim\limits_{x \to -\infty} e^x = 0$

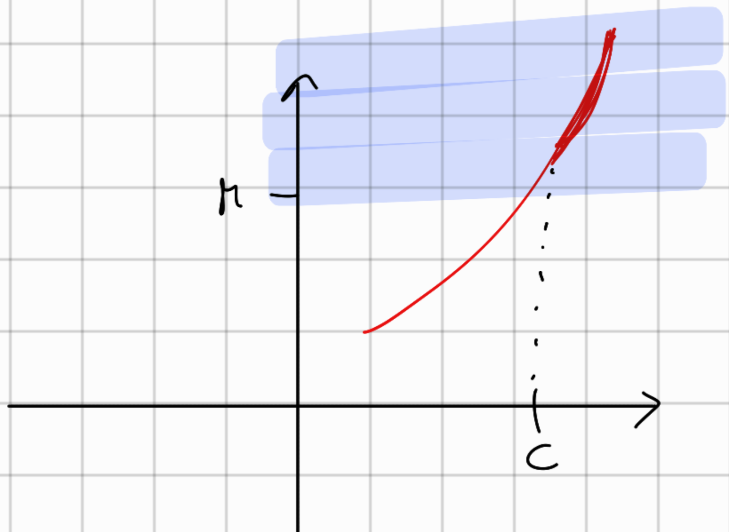
$e^{-5000} = \frac{1}{e^{5000}}$



DEFINIZIONE

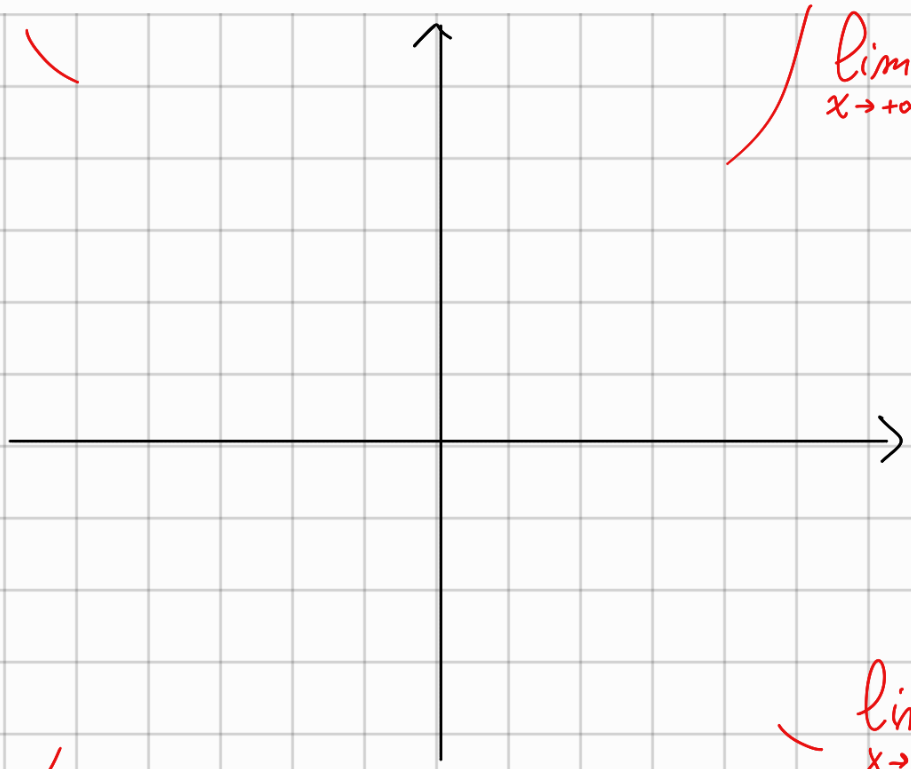
Diremo che $\lim\limits_{x \to +\infty\,(-\infty)} f(x) = +\infty \; (-\infty)$ (LA FUNZIONE DIVERGE)

$\forall M > 0 \; (<0) \quad \exists c > 0 \; (<0) \quad$ t.c. $\quad \forall x > c \; (<c) \quad f(x) > M \; (<M)$



$\lim_{x \to -\infty} f(x) = +\infty$

$\lim_{x \to +\infty} f(x) = +\infty$



$\lim_{x \to +\infty} f(x) = -\infty$

$\lim_{x \to -\infty} f(x) = -\infty$

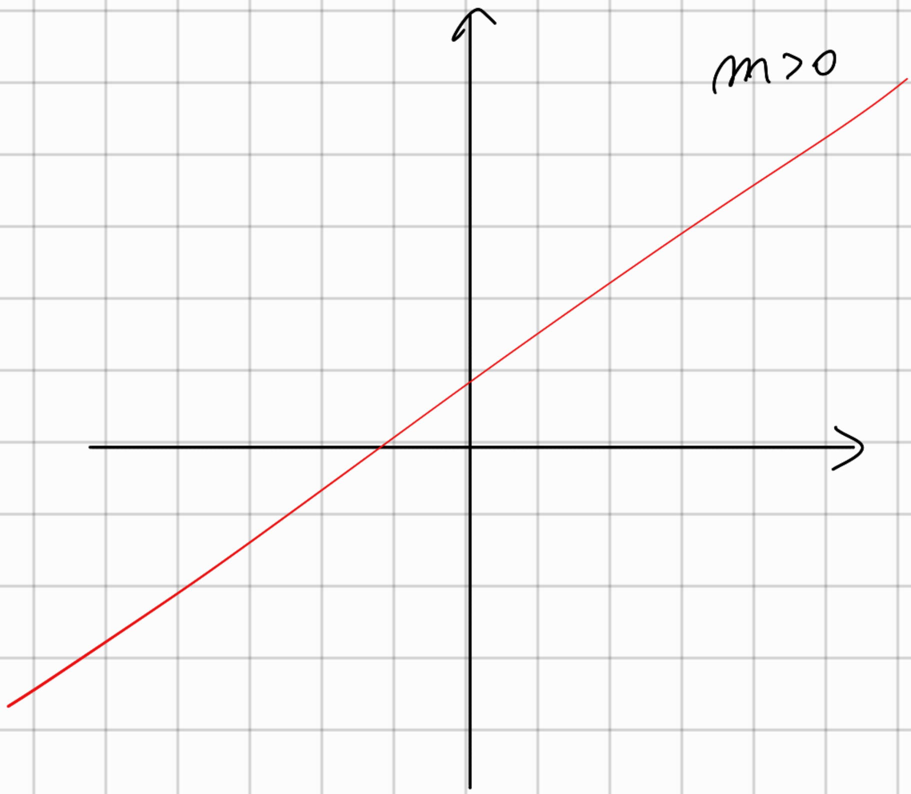
ESEMPI

$f(x) = mx + q \quad m \neq 0$

SE $m > 0$

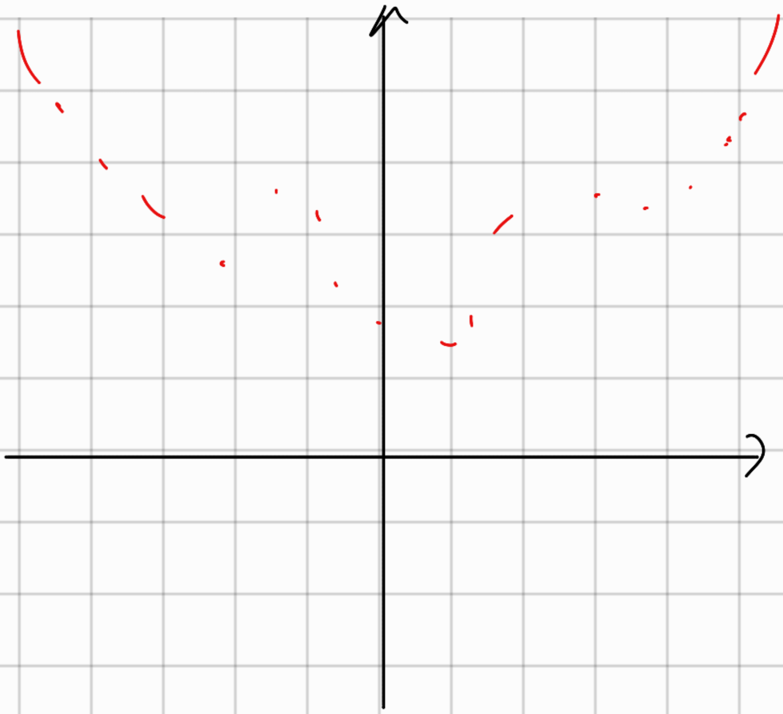
$\lim_{x \to +\infty} f(x) = +\infty$

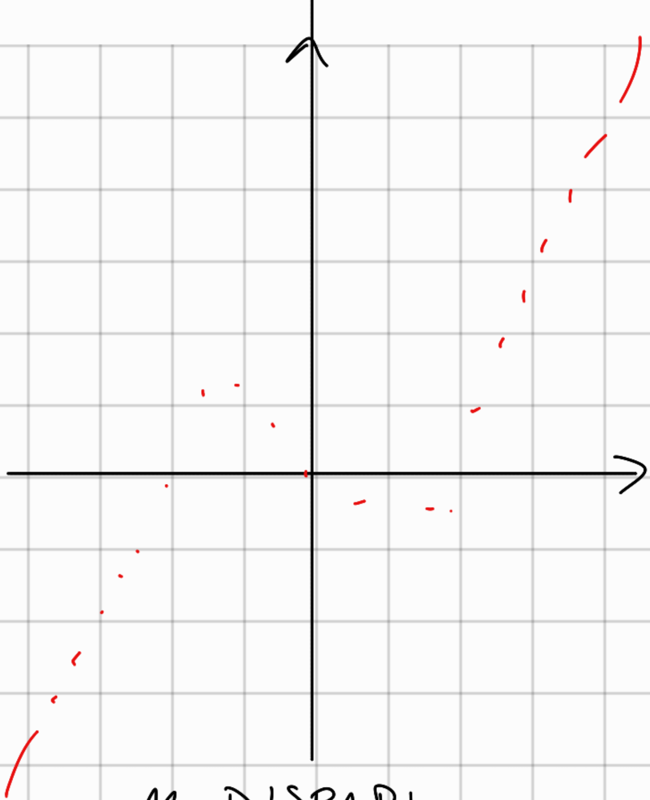
$\lim_{x \to -\infty} f(x) = -\infty$



PER I POLINOMI IN GENERALE VALE

$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \quad$ SE $a_n > 0$





n PARI

$$\lim_{x \to +\infty} P(x) = +\infty$$

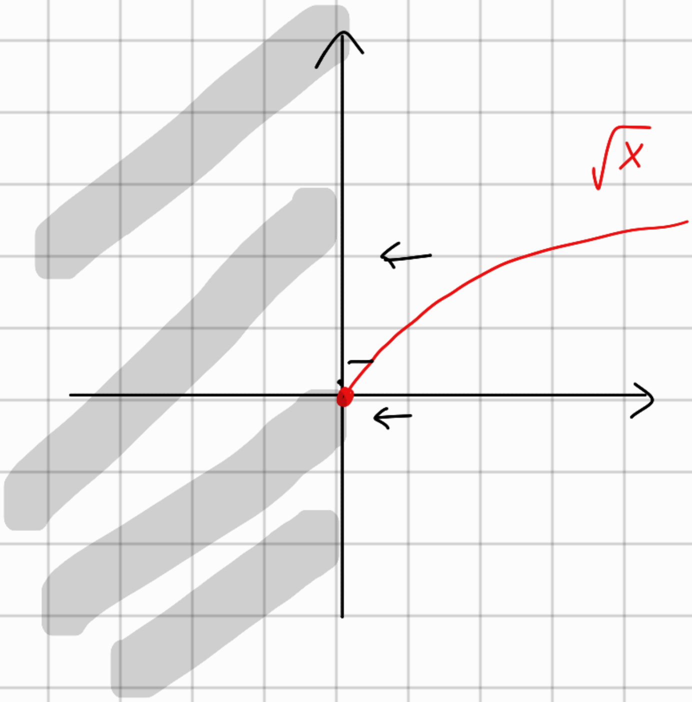
$$\lim_{x \to -\infty} P(x) = +\infty$$

n DISPARI

$$\lim_{x \to -\infty} P(x) = -\infty$$

$$\lim_{x \to +\infty} P(x) = +\infty$$

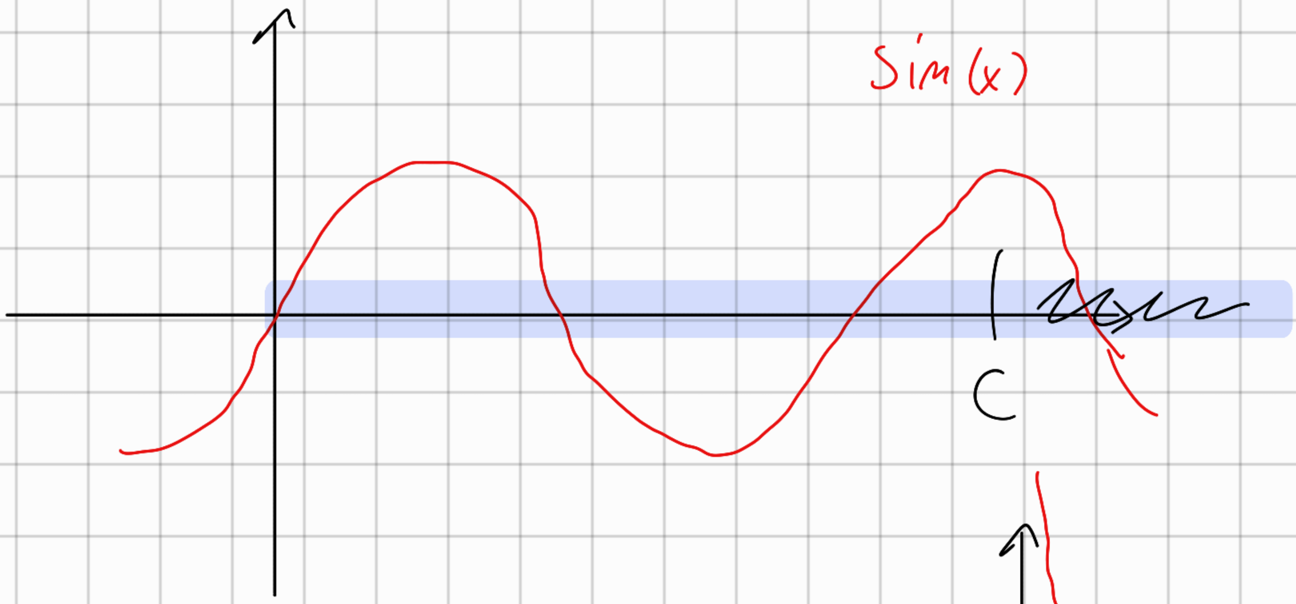
$$D = [0, +\infty)$$

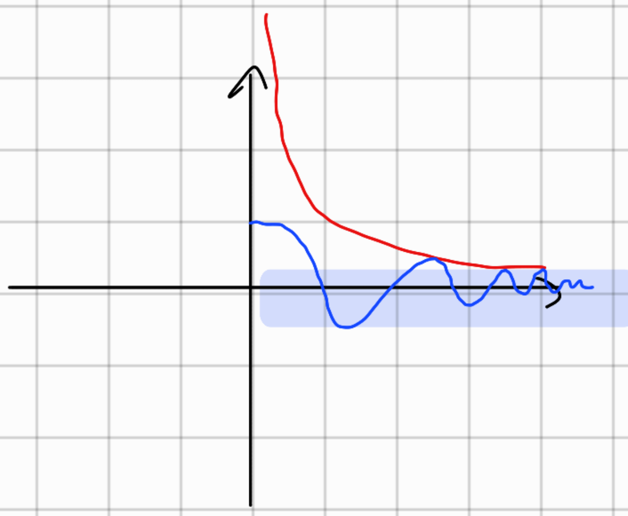


$$\lim_{x \to 0^+} \sqrt{x} = 0$$

$$\lim_{x \to +\infty} \sqrt{x} = +\infty$$

$\nexists \lim_{x\to\pm\infty} \sin x \ , \ \nexists \lim_{x\to\pm\infty} \cos x \quad \nexists \lim_{x\to\pm\infty} tg\, x$





$$\lim_{x\to+\infty} \frac{\sin x}{x} = 0$$