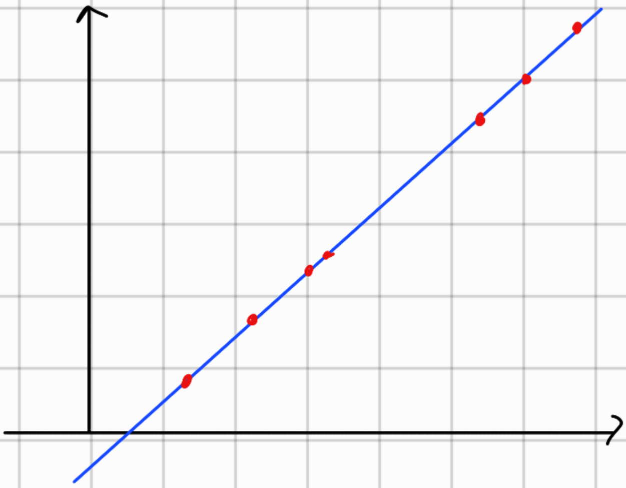
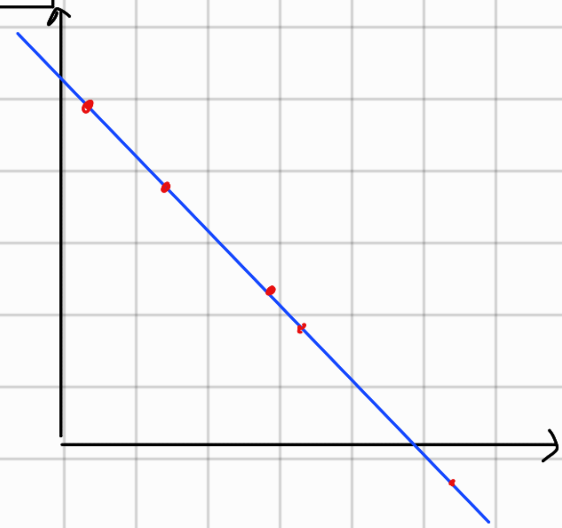
Lezione 31

16/12/24

CORRELAZIONE LINEARE $\boxed{\text{ESATTA}}$

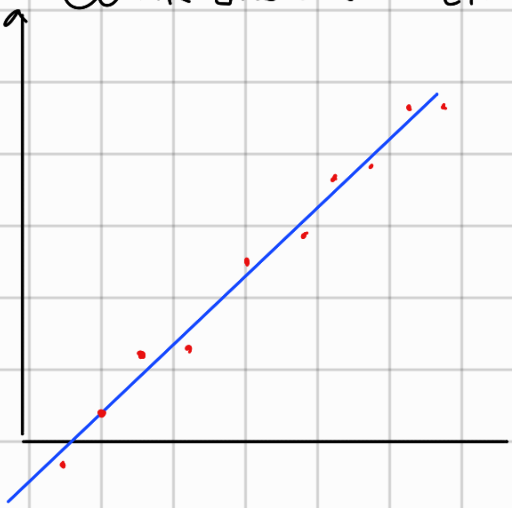


$\rho_{xy} = 1$

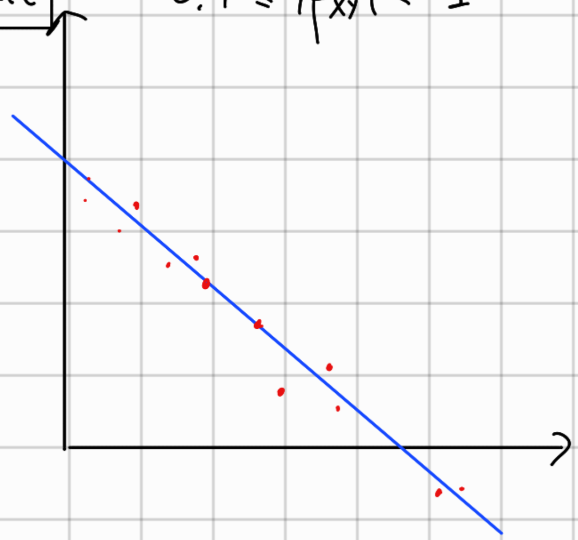


$\rho_{xy} = -1$

CORRELAZIONE LINEARE $\boxed{\text{FORTE}}$ $0.7 \leq |\rho_{xy}| < 1$

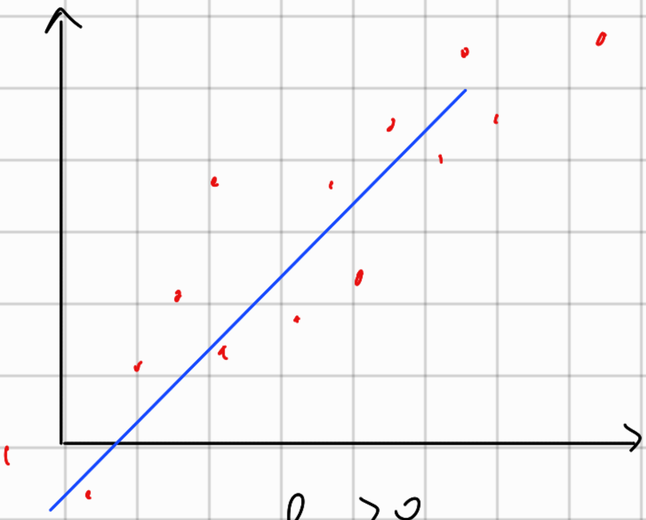


$\rho_{xy} > 0$

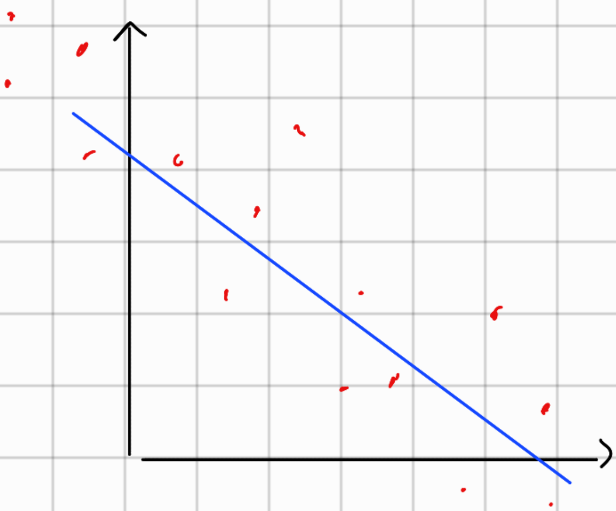


$\rho_{xy} < 0$

CORRELAZIONE LINEARE $\boxed{\text{MODERATA}}$ $0.3 \leq \rho_{xy} < 0.7$

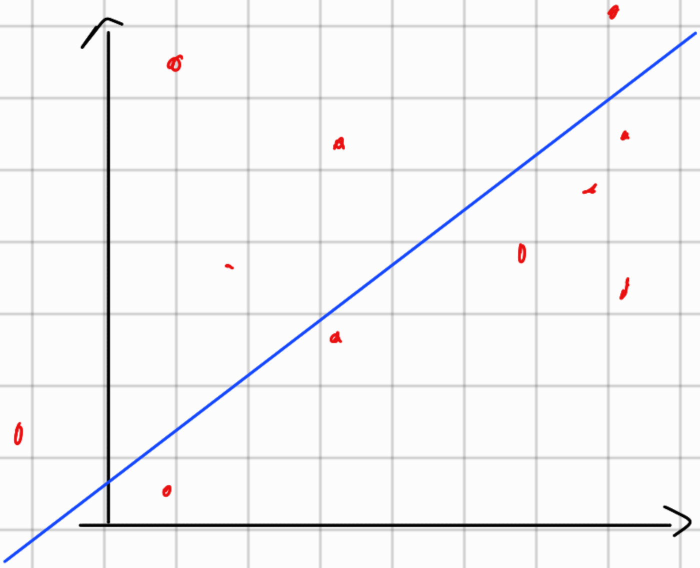


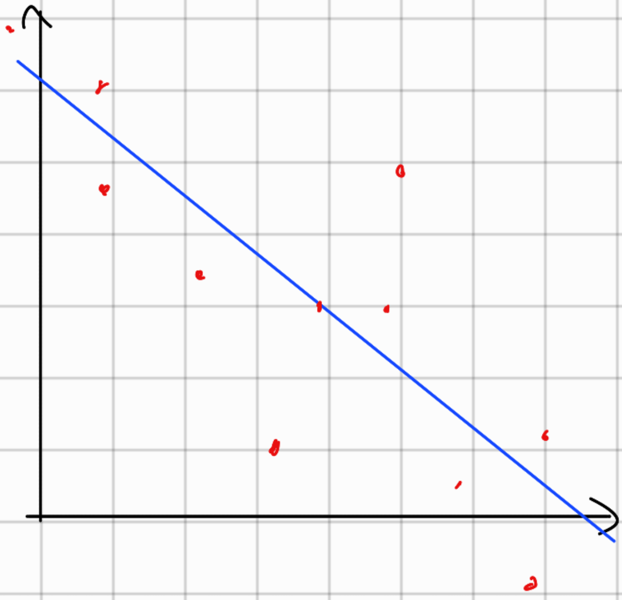
$\rho_{xy} > 0$



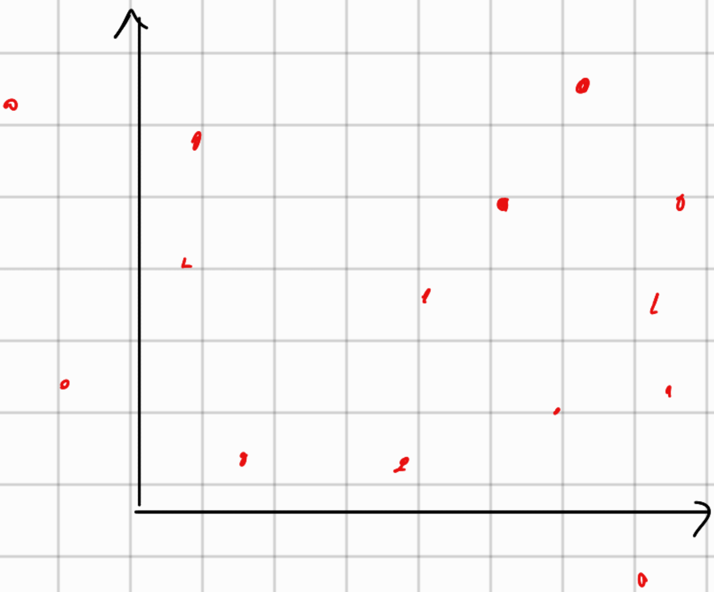
$\rho_{xy} < 0$

CORRELAZIONE LINEARE $\boxed{\text{DEBOLE}}$ $0 < |\rho_{xy}| < 0.3$





CORRELAZIONE LINEARE $\boxed{\text{NULLA}}$ $\rho_{xy} = 0$



ESEMPIO $\{(1,1), (5,9), (4,7), (10,19), (2,3)\}$

X	1	5	4	10	2
Y	1	9	7	19	3
XY	1	45	28	190	6

$N = 5$

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i = \frac{1}{5}(1+5+4+10+2) = \frac{22}{5} = 4.4$$

$$\bar{y} = \frac{1}{N}\sum_{i=1}^{N} y_i = \frac{1}{5}(1+9+7+19+3) = \frac{39}{5} = 7.8$$

$$\sigma_{xy} = \frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})(y_i-\bar{y}) = \frac{1}{5}\Big((1-4.4)(1-7.8)+(5-4.4)(9-7.8)+\dots+(2-4.4)(3-7.8)\Big)$$

$$\sigma_{xy} = \overline{xy} - (\bar{x}\bar{y}) = \frac{1}{5}(1+45+28+190+6) - (4.4\cdot 7.8) = 54-34.32 = 19.68$$

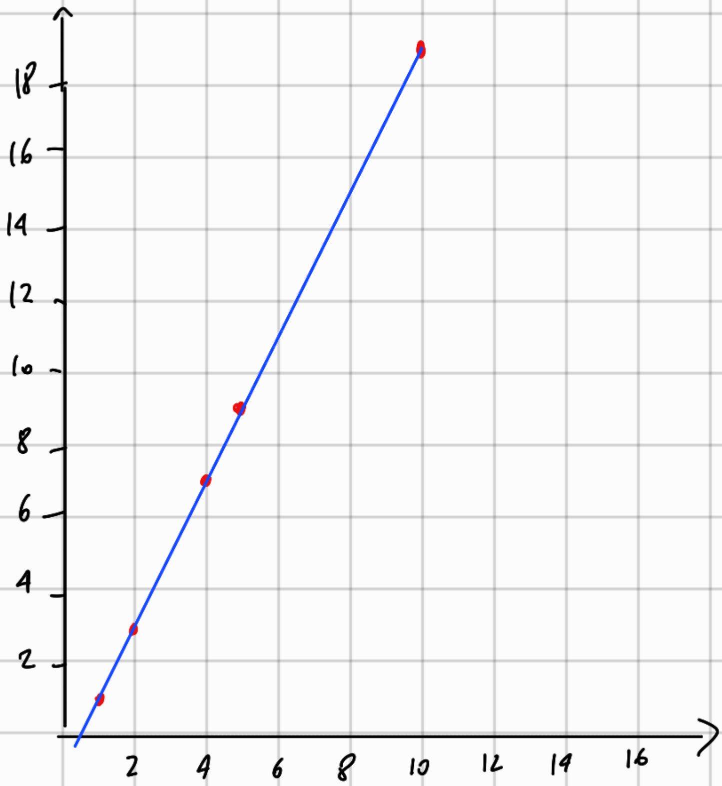
$$\sigma_x = \sqrt{\frac{1}{5}\left[(1-4.4)^2+(5-4.4)^2+(4-4.4)^2+(10-4.4)^2+(2-4.4)^2\right]} =$$

$$= \sqrt{\frac{1}{5}\left[(-3.4)^2+(0.6)^2+(-0.4)^2+(5.6)^2+(-2.4)^2\right]} = \sqrt{\frac{49.2}{5}} = \sqrt{9.84} \simeq 3.13$$

$$\sigma_y = \sqrt{\frac{1}{5}\left[(1-7.8)^2+(9-7.8)^2+(7-7.8)^2+(19-7.8)^2+(3-7.8)^2\right]} \simeq 6.27$$

$$\rho_{xy} = \frac{19.68}{3.13\cdot 6.27} = 1$$

CORRELAZIONE LINEARE ESATTA POSITIVA



$$\{(1,1),\ (5,9),\ (4,7),\ (10,19),\ (2,3)\}$$

I PUNTI SONO TUTTI ALLINEATI CERCO LA RETTA

$y = mx+q$ CHE PASSA PER DUE DI ESSI

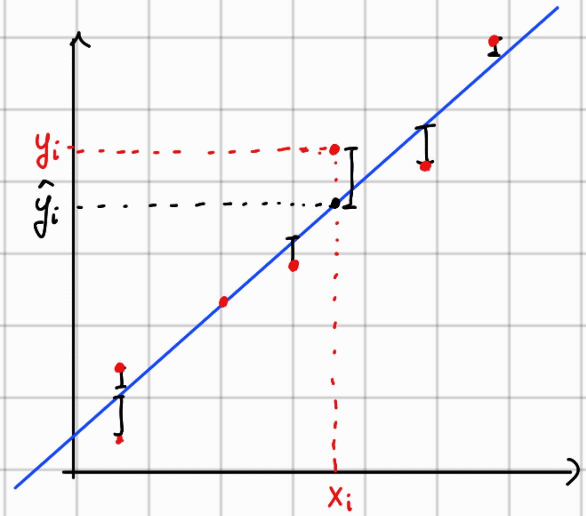
$$\begin{cases} 1 = 1m+q \\ 3 = 2m+q \end{cases} \rightarrow \begin{cases} q = 1-m \\ 3 = 2m+1-m \end{cases} \rightarrow \begin{cases} q = -1 \\ m = 2 \end{cases}$$

$$\boxed{y = 2x-1}$$

LA RETTA CHE APPROSSIMA MEGLIO I DATI SI CHIAMA

$\boxed{\text{RETTA DI REGRESSIONE LINEARE}}$

Utilizziamo il metodo dei minimi quadrati per calcolare α e β



$y = \alpha x + \beta$ $\quad \alpha, \beta$ INCOGNITE

$\hat{y}_i = \alpha x_i + \beta$ $\rightarrow$ APPROSSIMAZIONI

$\delta_i = |y_i - \hat{y}|$ ERRORE NEL PUNTO i

$$\min \underbrace{\left(\frac{1}{N} \sum_{i=1}^{N} (y_i - \hat{y})^2 \right)}_{J(\alpha, \beta)}$$

$\leftarrow$ QUESTA È UNA FUNZIONE CHE TIENE IN CONSIDERAZIONE TUTTI GLI ERRORI DI APPROSSIMAZIONE.

$J(\alpha, \beta)$ $\leftarrow$ FUNZIONE IN DUE VARIABILI CHE VOGLIAMO MINIMIZZARE

$$J(\alpha, \beta) = \frac{1}{N} \sum_{i=1}^{N} (y_i - \hat{y})^2 = \underbrace{\frac{1}{N} \sum_{i=1}^{N} \left[y_i - (\alpha x_i + \beta) \right]^2}$$

$$\min_{\alpha} \left[\min_{\beta} J(\alpha, \beta) \right]$$

VOGLIAMO CHE LA DERIVATA RISPETTO A β SIA NULLA (PENSANDO α COSTANTE)

DE J SU DE β

$$\frac{\partial J}{\partial \beta} = \frac{\partial}{\partial \beta} \left[\frac{1}{N} \sum_{i=1}^{N} [y_i - \alpha x_i - \beta]^2 \right] = \frac{1}{N} \sum_{i=1}^{N} 2 [y_i - \alpha x_i - \beta](-1) = 0$$

DERIVATA PARZIALE RISPETTO A β

$$\cancel{-2}\left[\frac{1}{N}\sum_{i=1}^{N} y_i - \alpha\frac{1}{N}\sum_{i=1}^{N} x_i - \frac{1}{N}\sum_{i=1}^{N}\beta\right] =$$

$$\frac{\partial}{\partial\beta}\left(f(\beta)\right)^2 = 2f(\beta)\cdot f'(\beta)$$

$$= \bar{y} - \alpha\bar{x} - \beta = 0$$

$$\frac{\beta+\beta+\beta}{3} = \frac{\cancel{3}\beta}{\cancel{3}} = \beta$$

$$\boxed{\beta = \bar{y} - \alpha\bar{x}}$$

$$\min_{\alpha} J(\alpha, \bar{y}-\alpha\bar{x}) = \min_{\alpha}\left[\frac{1}{N}\sum_{i=1}^{N}\left[y_i - (\alpha x_i + \bar{y} - \alpha\bar{x})\right]^2\right] =$$

$$y_i - \alpha x_i - \bar{y} + \alpha\bar{x} = y_i - \bar{y} - \alpha(x_i - \bar{x})$$

$$0 = \frac{\partial J}{\partial\alpha} = \frac{1}{N}\sum_{i=1}^{N}\cancel{2}\left[y_i - \bar{y} - \alpha(x_i-\bar{x})\right](x_i - \bar{x}) =$$

$$= \frac{1}{N}\sum_{i=1}^{N}\left[(y_i-\bar{y})(x_i-x) - \alpha(x_i-\bar{x})^2\right]$$

$$= \boxed{\frac{1}{N}\sum_{i=1}^{N}(y_i-\bar{y})(x_i-\bar{x})} - \alpha\boxed{\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})^2}$$

$$= \sigma_{xy} - \alpha\,\sigma_x^2$$

$$\boxed{\alpha = \frac{\sigma_{xy}}{\sigma_x^2}}$$

ESEMPIO: $N = 5$

Somministro una dose x_i di farmaco a 5 pazienti e osservo una diminuizione di pressione y_i, TROVO I SEGUENTI DATI:

X DOSE (mg)	Y DIMINUIZIONE (mmHg)
7	10
12	18
15	20
20	25
22	25

a) Scrivere le rette di regressione lineare.
b) Che dose dobbiamo dare se vogliamo una riduzione di circa 15 mmHg?
c) Di quanto diminuirà la pressione se viene somministrata una dose di 30 mmHg di farmaco?

CALCOLO

$$\begin{cases} \bar{x} = 15.2 \\ \bar{y} = 19.6 \\ \sigma_{xy} = 29.28 \\ \sigma_x^2 = 29.36 \end{cases}$$

$$\alpha = \frac{\sigma_{xy}}{\sigma_x^2} = \frac{29.28}{29.36} \simeq 0.997$$

$$\beta = \bar{y} - \alpha\bar{x} = 19.6 - 0.997 \cdot 15.2 \simeq 4.441$$

b) SE $y = 15$ mmHg

$$y = \alpha x + \beta \rightarrow y - \beta = \alpha x \rightarrow \boxed{x = \frac{y - \beta}{\alpha}}$$

$$x \simeq \frac{15 - 4.441}{0.997} \simeq 10.6 \text{ mg}$$

c) SE $X = 30$ mg

$$\boxed{y = \alpha x + \beta}$$

$$y \simeq (0.997)30 + 4.441 \simeq 34.4 \text{ mmHg}$$