Lezione 32 17/12/24

27 FEBBRAIO 2024 (PARI)

INTEGRAZIONE PER PARTI

$$\int f'g = \underbrace{fg} - \int fg'$$

2. $$\int \underbrace{x^2}_{f'} \underbrace{\ln x}_{g} \, dx =$$

$f = \frac{x^3}{3}$ $\quad g' = \frac{1}{x}$

$$= \frac{x^3}{3}\ln x - \int \frac{x^{\cancel{3}\,2}}{3} \cdot \frac{1}{\cancel{x}} dx = \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2 dx$$

$$= \frac{1}{3}\left[x^3 \ln x - \frac{x^3}{3}\right] + c = \underbrace{\frac{x^3}{3}\left[\ln x - \frac{1}{3}\right]}_{F(x)} + c$$

b. $$\int_1^e x^2 \ln x \, dx = \frac{x^3}{3}\left[\ln x - \frac{1}{3}\right]\Big|_1^e = \qquad e^0 = 1$$

$$= \underbrace{\frac{e^3}{3}\left[\underbrace{\ln e}_{=1} - \frac{1}{3}\right]}_{F(b)} - \underbrace{\frac{1}{3}\left[\underbrace{\ln 1}_{=0} - \frac{1}{3}\right]}_{F(a)} =$$

$$= \frac{e^3}{3}\left[1 - \frac{1}{3}\right] - \frac{1}{3}\left[-\frac{1}{3}\right] =$$

$$= \frac{e^3}{3}\left(\frac{2}{3}\right) + \frac{1}{9} = \frac{2}{9}e^3 + \frac{1}{9} = \frac{1}{9}\left[2e^3 + 1\right]$$

TEOREMA FONDAMENTALE DEL CALCOLO INTEGRALE

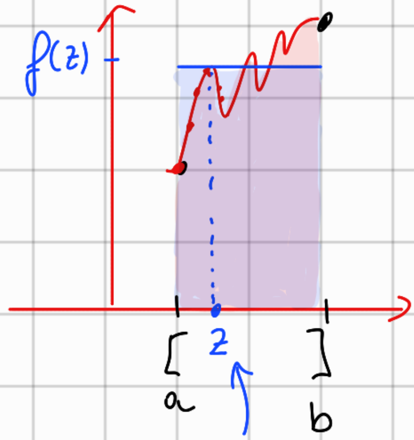
$$\int_a^b f(x) = F(b) - F(a)$$

$$F'(x) = f(x)$$

TEOREMA DELLA MEDIA INTEGRALE

Sia $f : [a,b] \to \mathbb{R}$ CONTINUA

$\exists\, z \in [a,b]$ t.c. $\int_a^b f(x)\,dx = (b-a) f(z)$



c. No perché l'intervallo $[1, e)$ NON È CHIUSO.

27 FEBBRAIO 2024 (PARI)

3. $$f(x) = \frac{3}{2}\ln x$$

$$\int f'g = \underbrace{fg} - \int fg'$$

a. $$\int \underbrace{\frac{3}{2}}_{f'} \underbrace{\ln x}_{g}\, dx = \frac{3}{2}x\ln x - \int \frac{3}{2}\cancel{x}\cdot\frac{1}{\cancel{x}}\,dx =$$

$f = \frac{3}{2}x$ $\quad g' = \frac{1}{x}$

$$= \frac{3}{2}x\ln x - \int \frac{3}{2}\,dx =$$

$c \in \mathbb{R}$

$$= \frac{3}{2}x\ln x - \frac{3}{2}x + c$$

$$= \underbrace{\frac{3}{2}x\left[\ln x - 1\right]}_{F(x)} + c$$

b. $$\int_1^e \underbrace{\frac{3}{2}\ln x}_{f(x)}\,dx = F(e) - F(1) = \frac{3}{2}e\left[\ln e - 1\right] - \frac{3}{2}\cdot 1\left[\ln 1 - 1\right] =$$

$$= \cancel{\frac{3}{2}e\left[1-1\right]} - \frac{3}{2}\left[-1\right] = \frac{3}{2}$$

c. Non può essere applicato perché il teorema richiede che l'intervallo sia chiuso.

27 FEBBRAIO 2024 (DISPARI)

ESERCIZIO 2

$$f(x) = \sin(2x)\, e^{\cos(2x)+\frac{1}{2}}$$

a. $$\int \sin(2x)\, e^{\cos(2x)+\frac{1}{2}}\, dx$$

$$\boxed{\cos(2x) = t}$$

$$-2\sin(2x)\,dx = dt$$

$$dx = \frac{1}{-2\sin(2x)}\,dt$$

$$\int \cancel{\sin 2x}\, e^{t+\frac{1}{2}} \left(\frac{1}{-2\cancel{\sin 2x}}\, dt \right)$$

$$\int e^{t+\frac{1}{2}} \left(-\frac{1}{2}\, dt \right)$$

$$-\frac{1}{2} \int e^{t+\frac{1}{2}}\, dt =$$

$$-2\sin(2x)\,dx = dt$$

$$\sin(2x)\,dx = -\frac{1}{2}\,dt$$

$$[e^{t+\frac{1}{2}}]' = e^{t+\frac{1}{2}}\,[t+\tfrac{1}{2}]' = e^{t+\frac{1}{2}}$$

$$-\frac{1}{2} e^{t+\frac{1}{2}} + C = -\frac{1}{2} e^{\cos(2x)+\frac{1}{2}} + C$$

$$\boxed{f(x) = t}$$

$$f'(x)\,dx = 1 \cdot dt$$

20 GIUGNO 2024 PARI

ESERCIZIO 2

$$\int \frac{3x+5}{\underbrace{x^2+2x-3}}\,dx = \int \frac{3x+5}{(x+3)(x-1)}\,dx = \dots$$

$\Delta = 4+12 = 16$

$$x_{12} = \frac{-2\pm 4}{2} \quad x_1 = \frac{-6}{2} = -3 \qquad x_2 = \frac{-2+4}{2} = 1$$

$A, B \in \mathbb{R}$

$$\boxed{\frac{3x+5}{(x+3)(x-1)} = \frac{A}{x+3} + \frac{B}{x-1}} = \frac{A(x-1)+B(x+3)}{(x+3)(x-1)} = \frac{Ax-A+Bx+3B}{(x+3)(x-1)}$$

$$= \frac{(A+B)x+3B-A}{(x+3)(x-1)}$$

$$\begin{cases} A+B=3 \\ 3B-A=5 \end{cases} \to \begin{cases} A=3-B \\ 3B-(3-B)=5 \end{cases} \to \begin{cases} A=3-B \\ 3B-3+B=5 \end{cases}$$

$$\begin{cases} '' \\ 4B=8 \end{cases} \to \begin{cases} A=3-2=1 \\ B=2 \end{cases}$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)|+c$$

$$\dots = \int\left(\frac{1}{x+3}+\frac{2}{x-1}\right)dx = \int \frac{1}{x+3}\,dx + 2\int \frac{1}{x-1}\,dx$$

$$= \underbrace{\ln|x+3| + 2\ln|x-1|}_{F(x)} + c$$

$$\int_{\underbrace{1+e}_{a}}^{\overbrace{5+e}^{b}} \frac{3x+5}{x^2+2x-3}\,dx = F(b)-F(a) =$$

$$= \ln|5+e+3| + 2\ln|5+e-1| - \left[\ln|1+e+3| + 2\ln|1+e-1|\right]$$

$$= \ln(8+e) + 2\underline{\ln(4+e)} - \underline{\ln(4+e)} - 2\ln(e)$$

$$= \ln(8+e) + \ln(4+e) - 2$$

$$\left(= \ln\left[(8+e)(4+e)\right] - 2\right)$$

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29 GENNAIO 2024 (DISPARI)

FASCIA D'ETÀ: V. QUALITATIVA ORDINATA
PROGRAMMA : V. QUALITATIVA NON ORDINATA.

	FILM	INFORMAZIONE	QUIZ		
giovani	22 $= F_{11}$	11 F_{12}	17 $= F_{13}$	50	$M_{1\cdot}$
adulti	13	37	30	80	$M_{2\cdot}$
anziani	25	12	33	70	$M_{3\cdot}$
	60	60	80	200	M $(M_{\cdot\cdot})$
	$\downarrow$ $M_{\cdot 1}$	$\downarrow$ $M_{\cdot 2}$	$\downarrow$ $M_{\cdot 3}$		

Posso eseguire il test perché $F_{ij} \geq 5$

E_{ij}	FILM	INFORMAZIONE	QUIZ		
giovani	15	15	20	50	$n_{1\cdot}$
adulti	24	24	32	80	$n_{2\cdot}$
anziani	21	21	28	70	$n_{3\cdot}$
	60	60	80	200	n $(n_{\cdot\cdot})$
	$n_{\cdot 1}$	$n_{\cdot 2}$	$n_{\cdot 3}$		

$$E_{11} = \frac{n_{1\cdot} \times n_{\cdot 1}}{n} = \frac{50 \cdot 60}{200} = \frac{300\cancel{0}}{20\cancel{0}} = 15$$

$k = 3$

$h = 3$

$(k-1)(h-1) = 2 \cdot 2 = 4$

$$\chi^{2\,*}_{(k-1)(h-1)} = \sum_{i=1}^{k} \sum_{j=1}^{h} \frac{(E_{ij} - F_{ij})^2}{E_{ij}}$$

$$\chi^{2\,*}_{4} = \frac{(15-22)^2}{15} + \frac{(15-11)^2}{15} + \frac{(20-17)^2}{20} + \frac{(24-13)^2}{24} + \frac{(24-37)^2}{24} + \frac{(32-30)^2}{32} +$$

$$+ \frac{(21-25)^2}{21} + \frac{(21-12)^2}{21} + \frac{(28-33)^2}{28} =$$

$$= \frac{49}{15} + \frac{16}{15} + \frac{9}{20} + \frac{121}{24} + \frac{169}{24} + \frac{4}{32} + \frac{16}{21} + \frac{81}{21} + \frac{25}{28} =$$

$$= 22.49 > 18.467$$

RIFIUTO L'IPOTESI H_0: LE DUE VARIABILI SONO INDIPENDENTI

CONCLUDO CHE AL 99.9 % IL PROGRAMMA TELEVISIVO e LA FASCIA D'ETA' SIANO INFLUENZATE TRA LORO.