

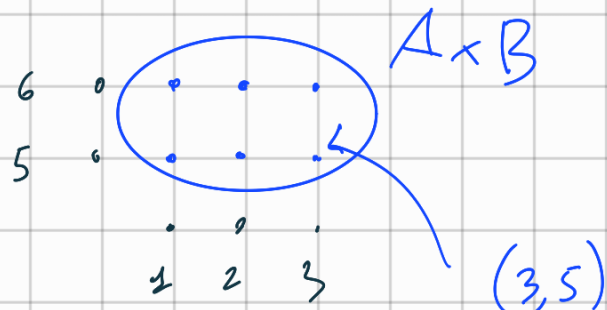
## Definizione

Sian  $A, B$  insiemi $\{a, b\}$  COPPIA $\{b, a\}$  $(a, b)$  COPPIA ORDINATA $A \times B = \{(a, b) : a \in A, b \in B\}$  PRODOTTO CARTESIANO

$$A = \{1, 2, 3\}$$

$$B = \{5, 6\}$$

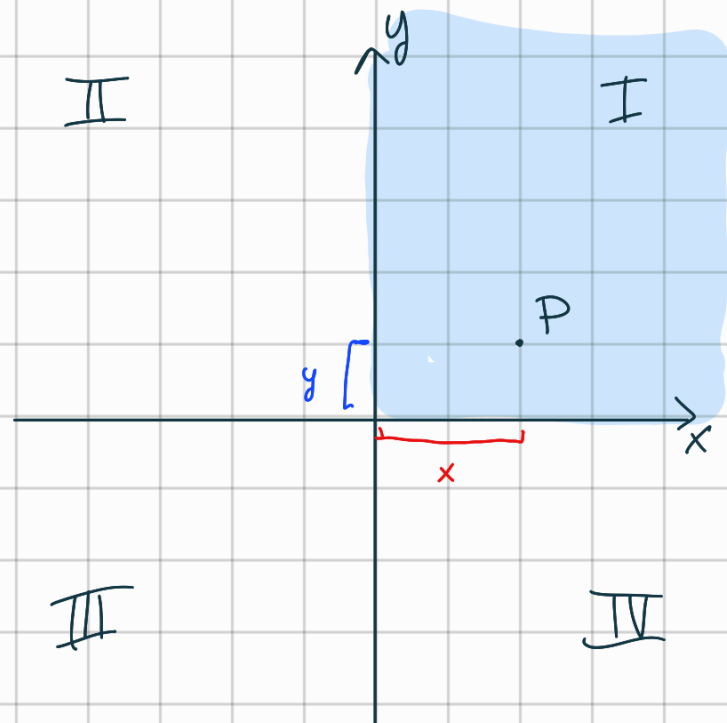
$$A \times B = \{(1, 5), (1, 6), (2, 5), (2, 6), (3, 5), (3, 6)\}$$



$$A = B = \mathbb{R}$$

$$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R} = \{(x, y) : a \in \mathbb{R}, b \in \mathbb{R}\}$$

$$P \in \mathbb{R}^2 \quad P = (x, y)$$



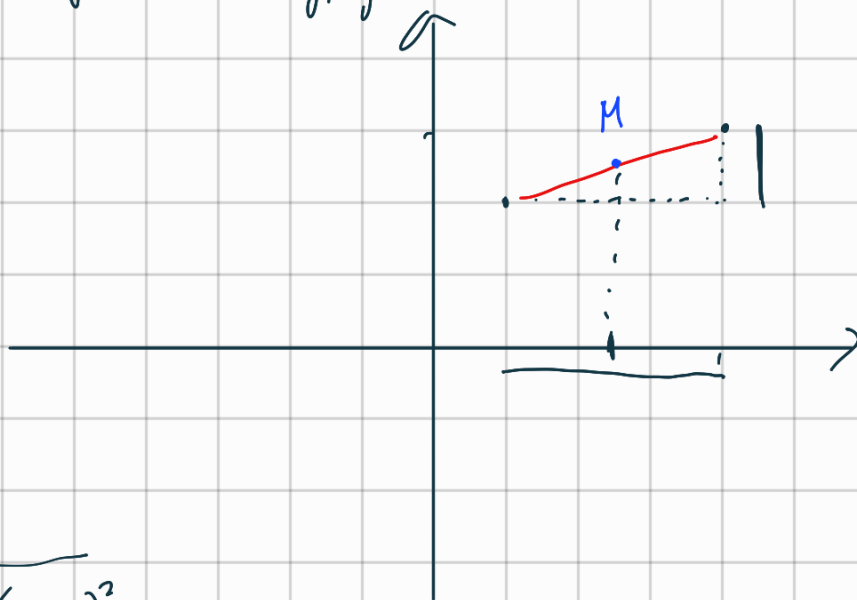
$$I = \{(x, y) \in \mathbb{R}^2 : x \geq 0, y \geq 0\} \subseteq \mathbb{R}^2$$

$$\text{dist} : \mathbb{R}^2 \times \mathbb{R}^2 \longrightarrow [0, +\infty)$$

$$(P_1, P_0) \longmapsto \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}$$

$$P_0 = (x_0, y_0)$$

$$P_1 = (x_1, y_1)$$



$$P_1 = (4, 3)$$

$$P_0 = (1, 2)$$

$$\text{dist}(P_1, P_0) = \sqrt{(4-1)^2 + (3-2)^2}$$

$$= \sqrt{3^2 + 1^2} = \sqrt{10}$$

Punto medio :  $\mathbb{R}^2 \times \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

$$(P_1, P_2) \longmapsto \left( \frac{x_0 + x_1}{2}, \frac{y_0 + y_1}{2} \right)$$

$$M = \left( \frac{4+1}{2}, \frac{3+2}{2} \right) = \left( \frac{5}{2}, \frac{5}{2} \right)$$

Definizione

$C = (x_0, y_0) \in \mathbb{R}^2$  CENTRO  
 $r \in [0, +\infty)$  RAGGIO

$$\gamma = \left\{ \underset{P}{(x, y)} \in \mathbb{R}^2 : \text{dist}(P, C) = r \right\}$$

$$\sqrt{(x-x_0)^2 + (y-y_0)^2} = r$$

$$(x-x_0)^2 + (y-y_0)^2 = r^2$$

$$x^2 - 2x_0x + x_0^2 + y^2 - 2y_0y + y_0^2 - r^2 = 0$$

$$x^2 + y^2 \underbrace{- 2x_0x}_a \underbrace{- 2y_0y}_b \underbrace{+ x_0^2 + y_0^2 - r^2}_c = 0$$

$$x^2 + y^2 + ax + by + c = 0$$

Se  $a, b, c \in \mathbb{R}$  t.c.  $a^2 + b^2 - 4c \geq 0$

$$\begin{cases} a = -2x_0 \rightarrow x_0 = -\frac{a}{2} \\ b = -2y_0 \rightarrow y_0 = -\frac{b}{2} \\ c = x_0^2 + y_0^2 - r^2 \end{cases}$$

$$r^2 = \frac{a^2}{4} + \frac{b^2}{4} - c$$

$$r = \sqrt{\frac{a^2}{4} + \frac{b^2}{4} - c}$$

$$\frac{a^2 + b^2 - 4c}{4} \geq 0$$

DATI  $a, b, c \in \mathbb{R} : a^2 + b^2 - 4c \geq 0$

$$f = \left\{ (x, y) \in \mathbb{R}^2 : x^2 + y^2 + ax + by + c = 0 \right\}$$

$$C = (1, 2)$$

$$r = 2$$

$$P \in f \quad P = (x, y)$$

$$\text{dist}(P, C) = 2$$

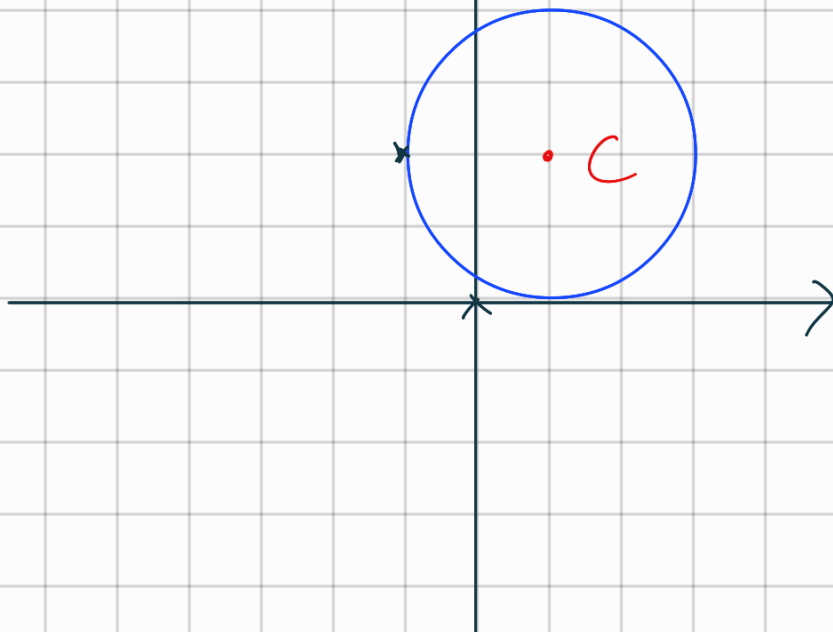
$$\sqrt{(x-1)^2 + (y-2)^2} = 2$$

$$(x-1)^2 + (y-2)^2 = 4$$

$$x^2 - 2x + 1 + y^2 - 4y + 4 = 4$$

$$x^2 + y^2 - 2x - 4y + 1 = 0$$

$$g: \begin{cases} a = -2 \\ b = -4 \\ c = 1 \end{cases}$$



$$\gamma = \{ (x,y) \in \mathbb{R}^2 : x^2 + y^2 - 2x - 4y + 1 = 0 \}$$

$$P_1 = (-1, 2) \in \gamma$$

$$(-1)^2 + (2)^2 - 2(-1) - 4(2) + 1 = 1 + 4 + 2 - 8 + 1 = 0 \quad \checkmark$$

$$P_2 = (0, 0) \notin \gamma$$

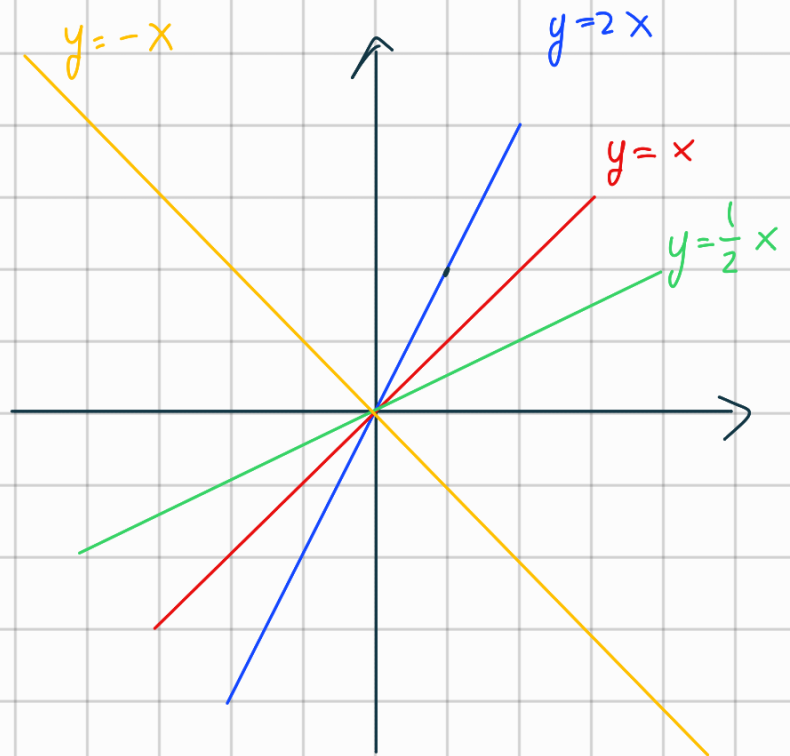
$$0 + 0 + 0 + 0 + 1 \neq 0$$

$$r = \{ (x,y) : y = 2x \}$$

Scelgo  $m \in \mathbb{R}$

$$y = mx \quad \begin{matrix} m \\ \text{COEFFICIENTE} \\ \text{ANGOLARE} \end{matrix}$$

$$x = 0$$

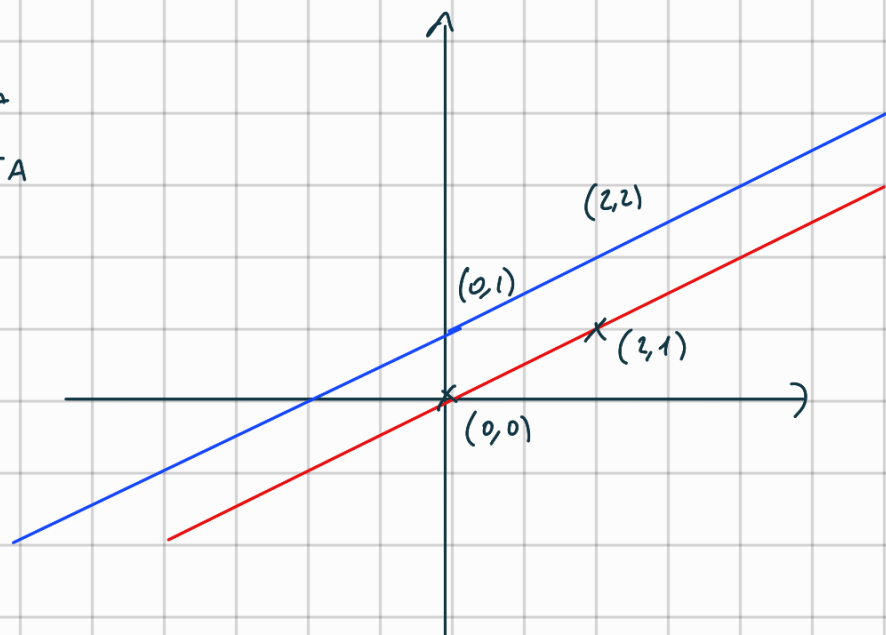


$q \in \mathbb{R}$  INTERCETTA

$$y = mx + q$$

EQ DI UNA RETTA  
FORMA ESPLICITA

$$x = q \quad \text{RETTE VERTICALI}$$



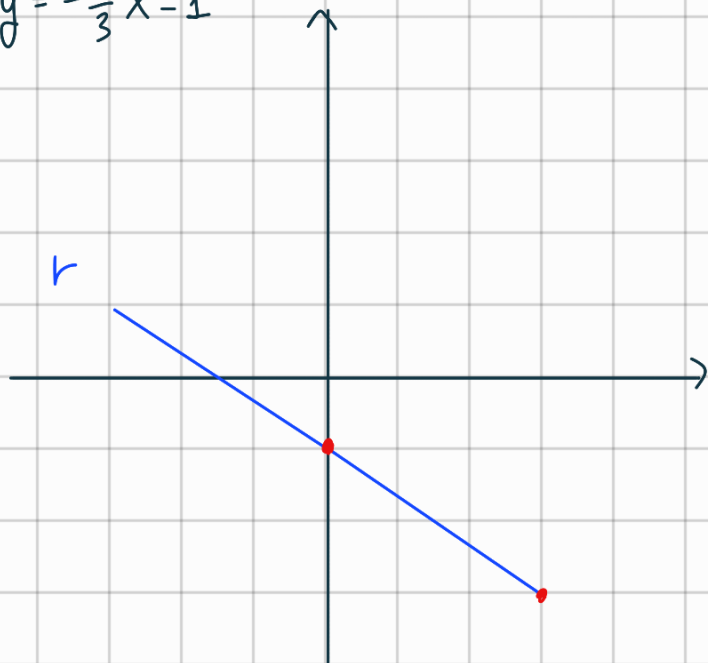
Disegnare la retta  $r: y = -\frac{2}{3}x - 1$

$$x = 0 \Rightarrow y = -1$$

$$x = 3 \Rightarrow y = -\frac{2}{3} \cdot 3 - 1 = -3$$

$$P_1 = (0, -1)$$

$$P_2 = (3, -3)$$



$$f: \mathbb{R} \rightarrow \mathbb{R}$$

$$x \mapsto y = 5x + 2$$

Esempio Viaggio a  $10 \frac{\text{km}}{\text{h}}$  parto dal km 2

A quale chilometro mi trovo dopo mezz'ora?

$$y = 10x + 2$$

$x$  è le ore di viaggio  
 $y$  è il km in cui mi trovo  
(le mie posizioni)

$$y = 10 \cdot \frac{1}{2} + 2 = 7$$

DIRETTAMENTE PROPORZIONALE



$$y = mx$$



$$ax + by + c = 0$$

è UNA RETTA

$$\forall a, b, c \in \mathbb{R}$$

ALMENO UNO FRA  $a$  E  $b$  DEVE ESSERE DIVERSO DA 0

EQUAZIONE IMPLICITA

$$\begin{cases} y = -\frac{a}{b}x - \frac{c}{b} \\ x = -\frac{c}{b} \end{cases}$$

se  $b \neq 0$

$$m = -\frac{a}{b}$$

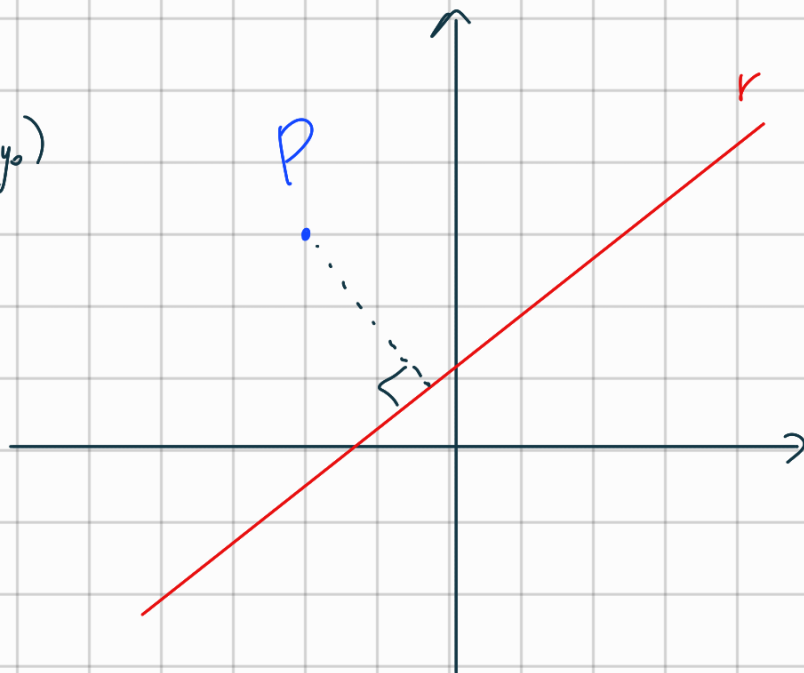
$$q = -\frac{c}{b}$$

&  $b = 0$

$\mathbb{R} = \{ \text{rette del piano} \}$

$r: ax + by + c \quad P = (x_0, y_0)$

$$\text{dist}(P, r) = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$$



VALORE ASSOLUTO

$$|\cdot| : \mathbb{R} \rightarrow \mathbb{R}$$

$$x \mapsto \begin{cases} x & \text{se } x \geq 0 \\ -x & \text{se } x < 0 \end{cases}$$

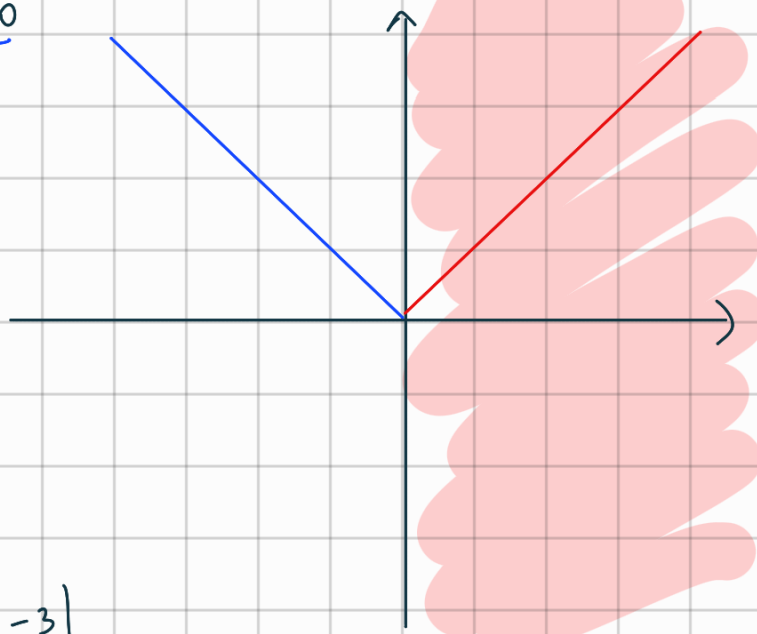
$$|3| = 3$$

$$|-3| = 3$$

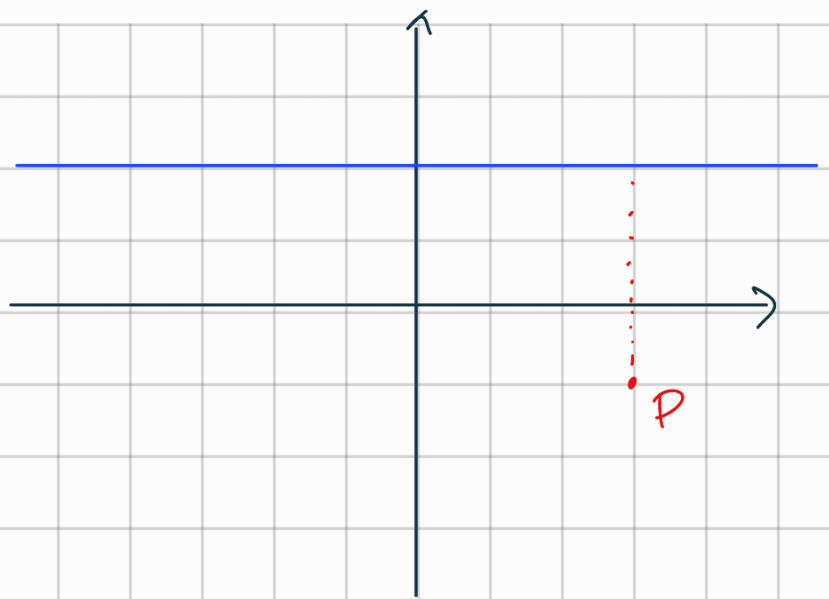
$$r: y - 2 = 0 \quad \begin{cases} a = 0 \\ b = 1 \\ c = -2 \end{cases}$$

$$P = (3, -1)$$

$$\text{dist}(P, r) = \frac{|0 + (1)(-1) - 2|}{\sqrt{0^2 + 1^2}} = \frac{|-3|}{1} = 3$$



$$y=2$$



Definizione

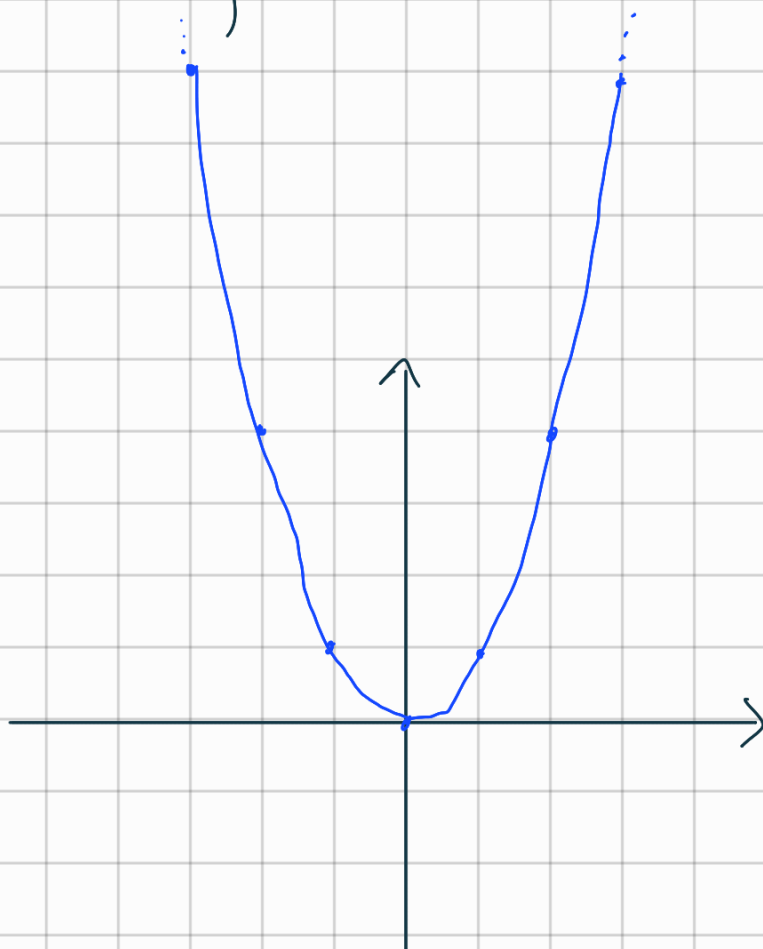
Sia  $f: A \subseteq \mathbb{R} \rightarrow \mathbb{R}$  funzione il GRAFICO di  $f$  è

$$\Gamma_f = \{(x, f(x)) \in \mathbb{R}^2 : x \in A\}$$

ESEMPIO

$$f(x) = x^2$$

$$\Gamma_f = \{(x, x^2) : x \in \mathbb{R}\}$$



La circonferenza non  
è una funzione

