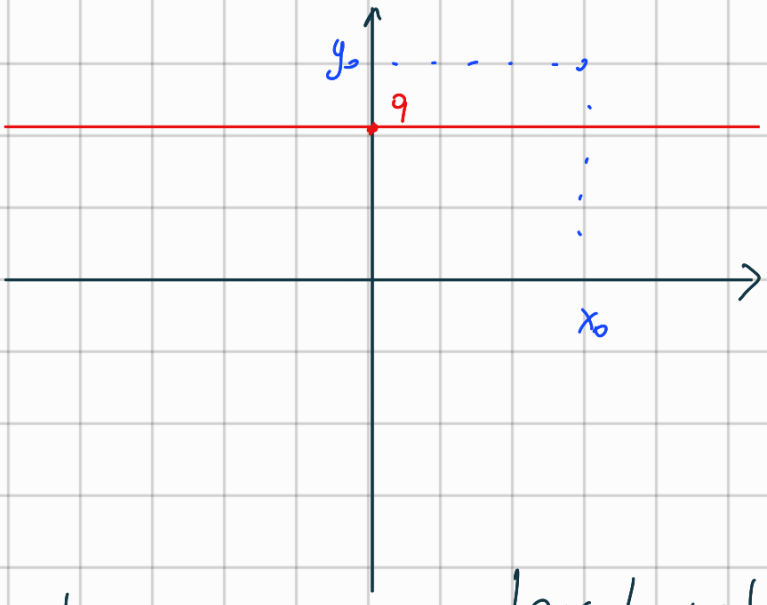
Lezione 6

06/11/23

$0x + 1y - q = 0$

$d: \quad y - q = 0$

$F: \quad (x_0, y_0) \in \mathbb{R}^2$



Sia $P \in$ PARABOLA $\quad P = (x, y)$

$$dist(F, P) = dist(d, P)$$

$$\sqrt{(x - x_0)^2 + (y - y_0)^2} = \frac{|y - q|}{\sqrt{1}}$$

$$\frac{|ax + by + c|}{\sqrt{a^2 + b^2}}$$

$$(x - x_0)^2 + (y - y_0)^2 = (y - q)^2$$

$$|y - q|^2 = (y - q)^2$$

$$x^2 - 2x_0x + x_0^2 + y^2 - 2y_0y + y_0^2 = y^2 - 2qy + q^2$$

$$-2y_0y + 2qy = -x^2 + 2x_0x - x_0^2 - y_0^2 + q^2$$

$$-2(y_0 - q)y = -x^2 + 2x_0x - x_0^2 - y_0^2 + q^2$$

$y_0 \neq q$

$$y = \underbrace{\frac{1}{2(y_0 - q)}}_{a} x^2 - \underbrace{\frac{x_0}{y_0 - q}}_{b} + \underbrace{\frac{x_0^2 + y_0^2 - q^2}{2(y_0 - q)}}_{c}$$

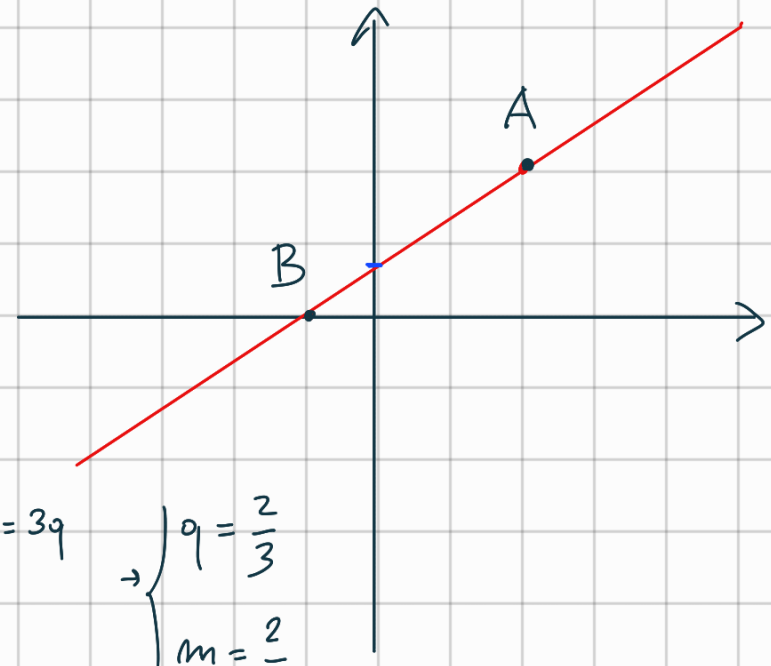
Disegnare la retta che passa per due punti: $A=(2,2)$, $B=(-1,0)$

$$y = mx + q$$

$$\begin{cases} 2 = m(2) + q \\ 0 = m(-1) + q \end{cases}$$

$$\downarrow$$

$$\begin{cases} 2 = 2m + q \\ 0 = -m + q \end{cases} \rightarrow \begin{cases} 2 = 2q + q \\ m = q \end{cases} \rightarrow \begin{cases} 2 = 3q \\ \end{cases} \rightarrow \begin{cases} q = \frac{2}{3} \\ m = \frac{2}{3} \end{cases}$$



CIRCONF. CHE PASSA PER $(3,0)$, $(1,2)$, $(0,-\sqrt{3})$

$$x^2 + y^2 + ax + by + c = 0$$

$$\begin{cases} 9 + 3a + c = 0 \\ 1 + 4 + a + 2b + c = 0 \\ 3 + -\sqrt{3}b + c = 0 \end{cases} \rightarrow \begin{cases} 3a + c = -9 \\ a + 2b + c = -5 \\ -\sqrt{3}b + c = -3 \end{cases} \rightarrow \begin{cases} c = -9 - 3a \\ a + 2b - 9 - 3a = -5 \\ \sim \end{cases}$$

$$\begin{cases} \sim \\ a - b = -2 \\ -2a + 2b = 4 \\ \sim \end{cases} \rightarrow \begin{cases} \sim \\ a = b - 2 \\ -\sqrt{3}b - 9 - 3(b-2) = -3 \end{cases} \begin{cases} c = -9 + 6 = -3 \\ a = -2 \\ b = 0 \end{cases}$$

$$-\sqrt{3}b - 9 - 3b + 6 = -3$$

$$-(3+\sqrt{3})b = 0$$

$$b = 0$$

L'eq. della circonferenza è $x^2 + y^2 - 2x - 3 = 0$

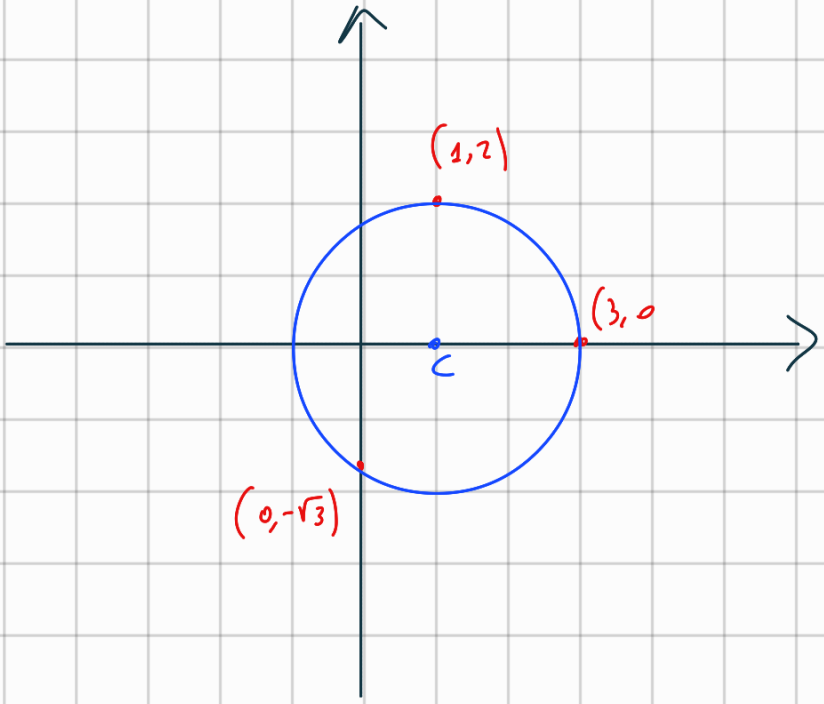
$$x^2 + y^2 - 2x - 3 + 1 - 1 = 0$$

$$(x-x_c)^2 + (y-y_c)^2 = r^2$$

$$(x-1)^2 + (y-0)^2 = 4$$

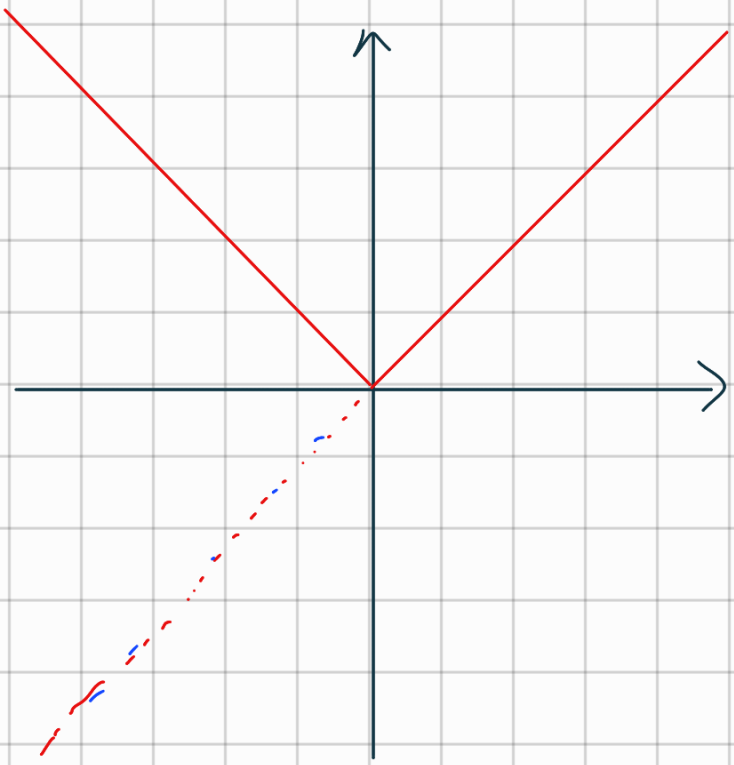
$C = (1, 0)$

$r = 2$



$$|\cdot| : \mathbb{R} \longrightarrow \mathbb{R}$$

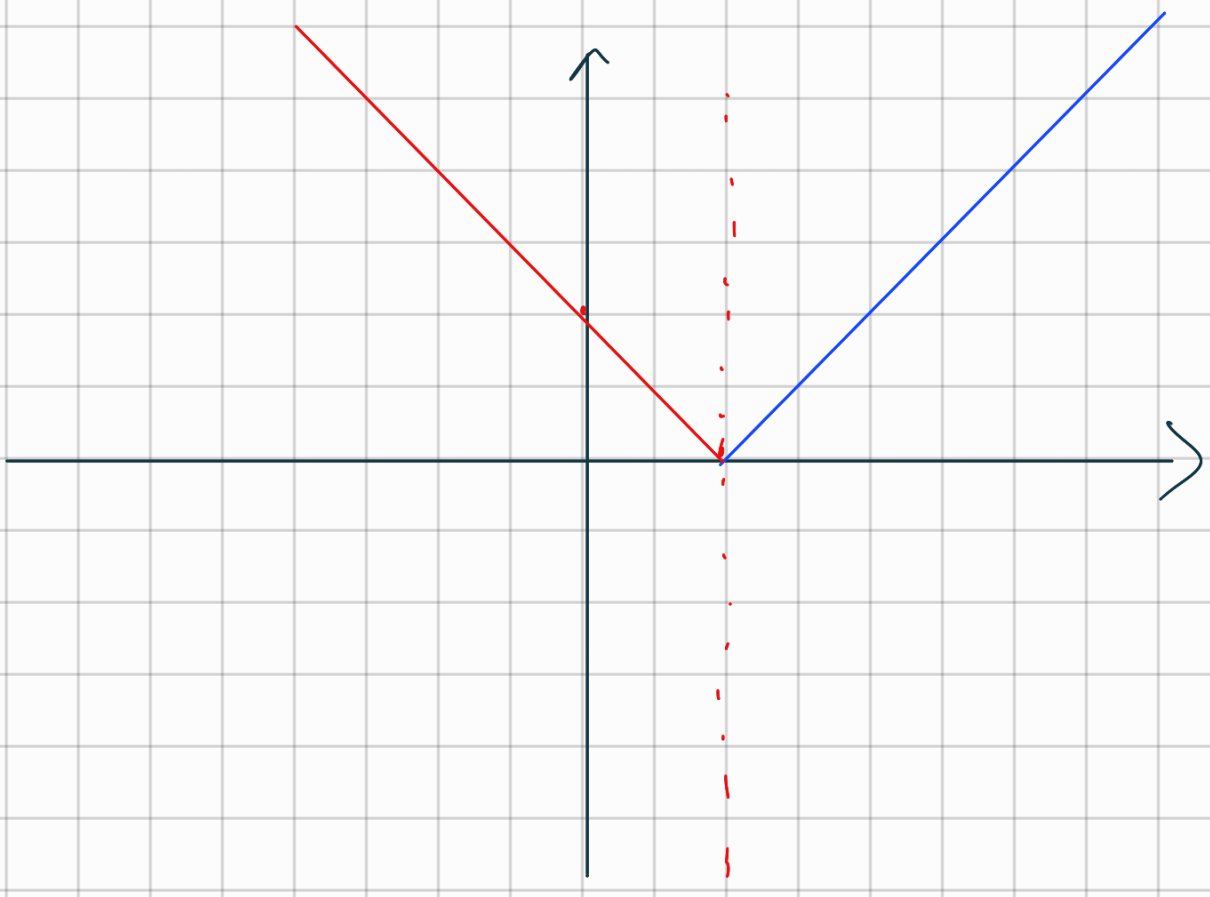
$$x \longmapsto \begin{cases} x & \text{se } x \geq 0 \\ -x & \text{se } x < 0 \end{cases}$$

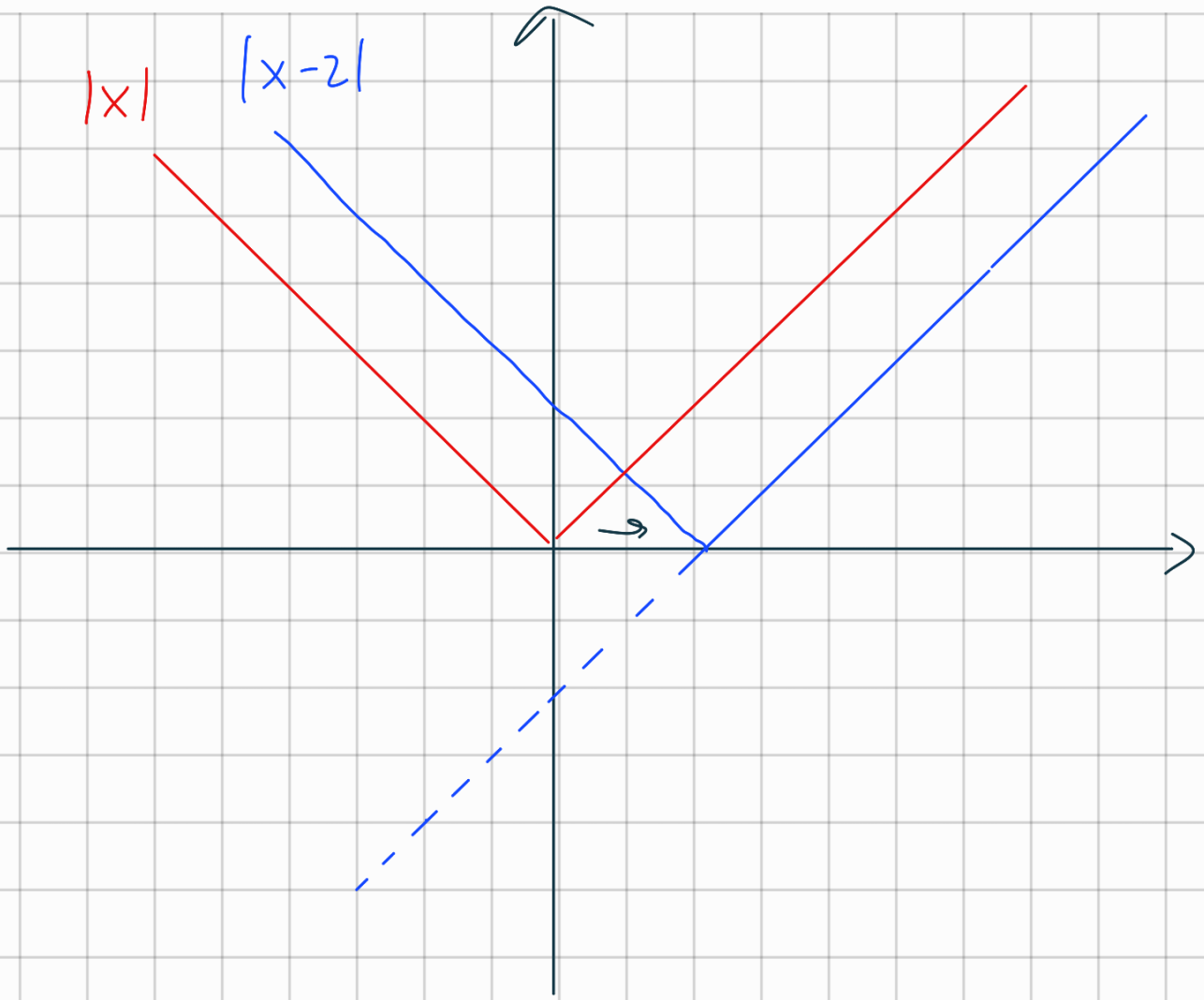


Disegnare $|x-2|$

$$|x-2| = \begin{cases} x-2 & \text{se } x-2 \geq 0 \\ -x+2 & \text{se } x-2 < 0 \end{cases}$$

$$= \begin{cases} x-2 & \text{se } x \geq 2 \\ -x+2 & \text{se } x < 2 \end{cases}$$





Disegnare $|x^2-4|$

$$\begin{cases} x^2-4 & \text{se } x^2-4 \geq 0 \\ -x^2+4 & \text{se } x^2-4 < 0 \end{cases}$$

$x^2-4 \geq 0$ CONCORDI

calcolo le sol. dell'eq. associata

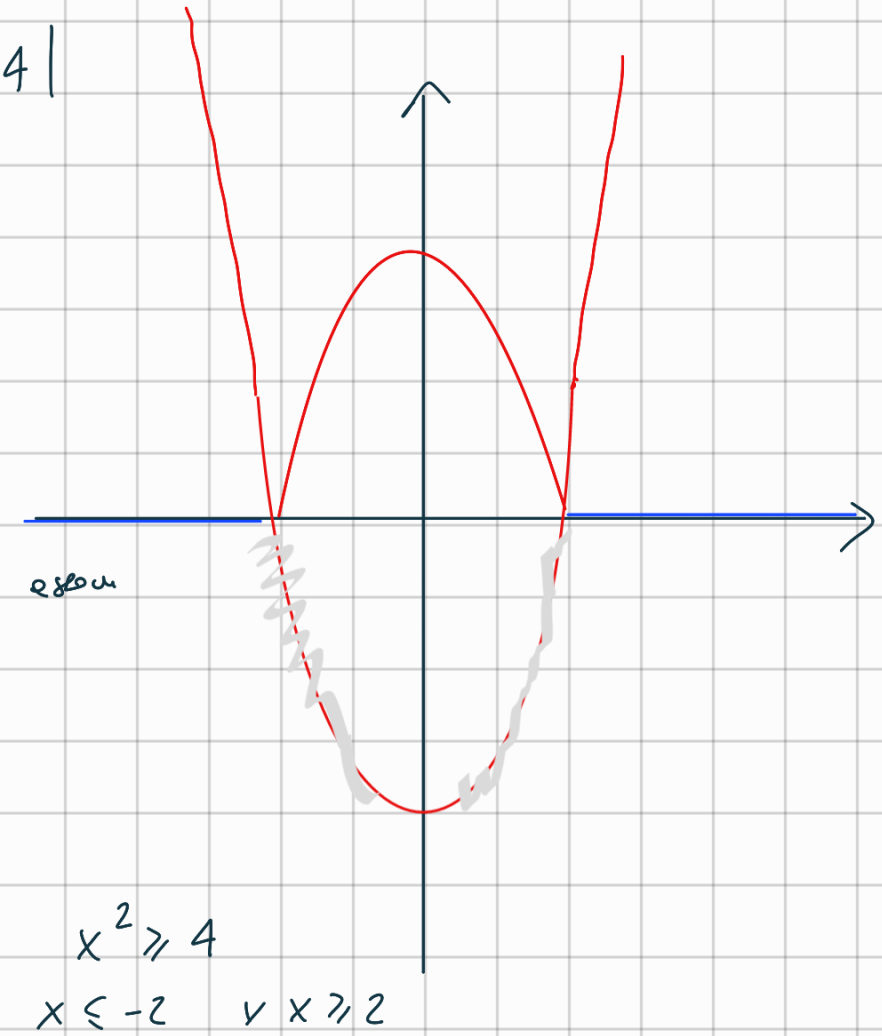
$x^2-4=0$

$x_1=-2$

$x_2=2$

$x \leq -2 \vee x \geq 2$

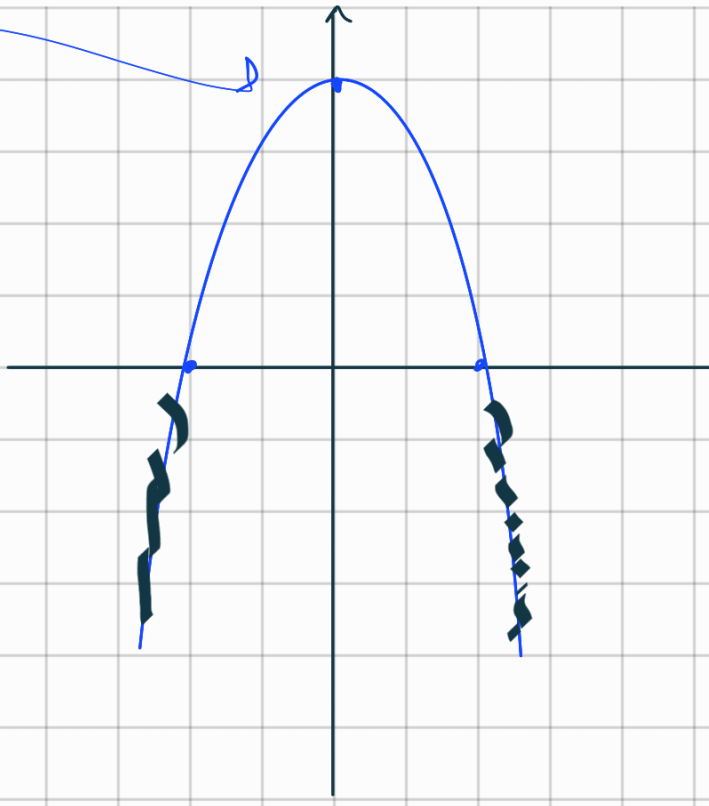
$(-\infty,-2] \cup [2,+\infty)$



$x^2 \geq 4$

$x \leq -2 \quad \vee \quad x \geq 2$

$-x^2+4$ è una parabola



$x_1 = -2$

$x_2 = 2$

TRASLAZIONI A DX e A SX

Se ho $y = f(x)$ funzione $f(x-x_0)$ è simile e $f(x)$ ma traslata di x_0 verso DESTRA

Disegna la funzione $|x^2-4|$ traslata di 1 verso l'alto
di 2 verso sinistra

~~$|x^2+2-4|+1$~~

$|(x+2)^2-4|+1$

$|x^2+4x+4-4|+1$

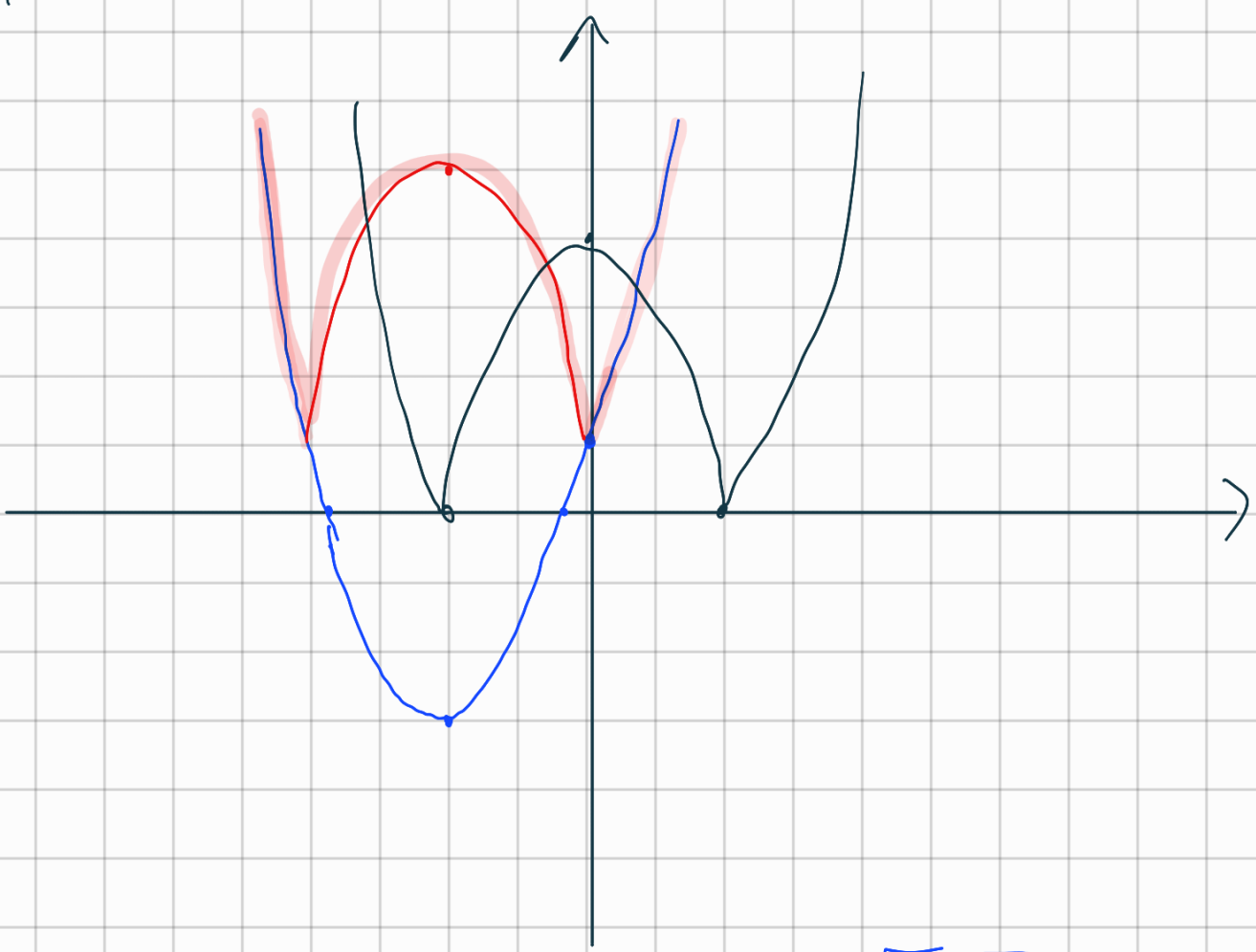
$|x^2+4x|+1 = \begin{cases} x^2+4x+1 & \text{se } x^2+4x \geq 0 \\ -x^2-4x+1 & \text{se } x^2+4x < 0 \end{cases}$

$x^2+4x \geq 0$

$x(x+4)=0 \quad \nearrow x_1=-4 \quad \searrow x_2=0$

$(-\infty,-4]\cup[0,+\infty)$

$$\begin{cases} x^2+4x+1 & \text{se} \quad x \in (-\infty,-4]\cup[0,+\infty) \\ -x^2-4x+1 & \text{se} \quad x \in (-4,0) \end{cases}$$



$\sqrt{12}=\sqrt{9\cdot 3}=\sqrt{4}\cdot\sqrt{3}=2\sqrt{3}$

x^2+4x+1

$\Delta = 16-4=12$

$x_{1,2}=\dfrac{-4\pm\sqrt{12}}{2} \quad \nearrow \dfrac{-4-2\sqrt{3}}{2}=-2-\sqrt{3}\simeq -3.6 \quad \searrow \dfrac{-4+2\sqrt{3}}{2}=-2+\sqrt{3}\simeq -0.4$

$1=\sqrt{1}<\sqrt{3}<\sqrt{4}=2$

$1<\sqrt{3}<2$

$$V = \left(-\frac{b}{2a}, -\frac{\Delta}{4a}\right) = \left(-\frac{4}{2}, -\frac{12}{4}\right) = (-2, -3)$$

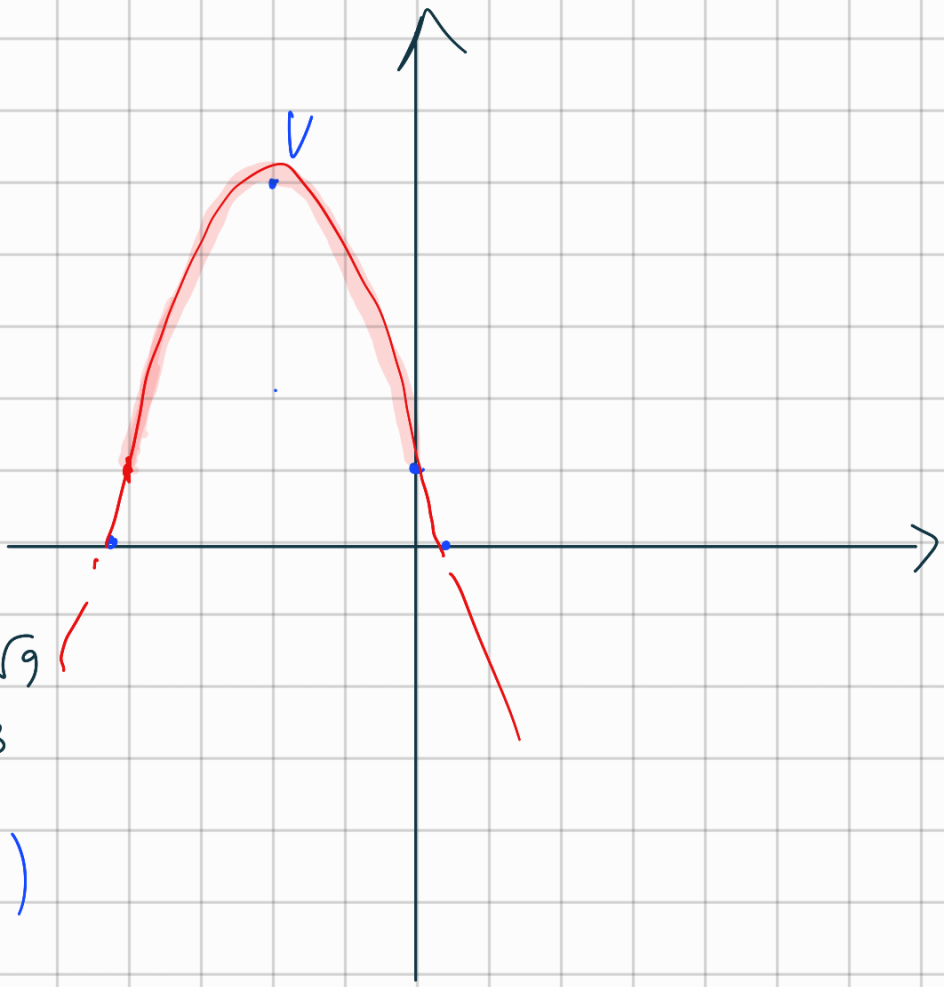
$$-x^2 - 4x + 1$$

$$\Delta = 16 + 4 = 20$$

$$x_{1,2} = \frac{4 \pm 2\sqrt{5}}{-2}$$

$$x_1 = -2 - \sqrt{5}$$

$$x_2 = -2 + \sqrt{5}$$



$$\sqrt{4} < \sqrt{5} < \sqrt{9}$$

$$2 < \sqrt{5} < 3$$

$$V = \left(\frac{4}{-2}, \frac{-20}{-4}\right) = (-2, 5)$$

$$\sqrt{4 \cdot 5} = \sqrt{4} \cdot \sqrt{5} = 2\sqrt{5}$$

Disequazioni fratte

$> \quad \geq \quad < \quad \leq$

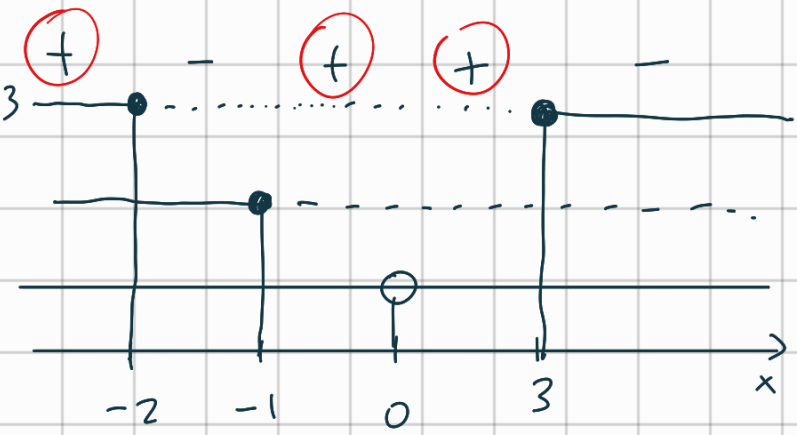
$$\frac{f(x)\,g(x)}{h(x)} \geq 0 \Rightarrow S$$

$f(x) \geq 0 \rightarrow x \leq -2 \vee x \geq 3$

$g(x) \geq 0 \rightarrow x \leq -1$

$h(x) > 0 \rightarrow \mathbb{R} \setminus \{0\}$

NON LO METTO PERCHÉ IL DEN $\neq 0$



$$S = (-\infty, -2) \cup (-1, 0) \cup (0, 3)$$

$$\frac{x^2+25}{x^2-4x} \leq 0$$

$x^2+25 > 0$

$\Delta = -100 < 0$

$S_1 = \mathbb{R}$

$x^2 - 4x > 0$

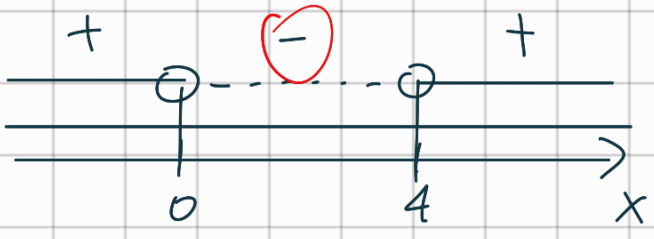
eq. associata

$x(x-4) = 0$

$x_1 = 0, \quad x_2 = 4$

$S_2 = (-\infty, 0) \cup (4, +\infty)$

$\{x \in \mathbb{R} : x < 0 \vee x > 4\}$



$S = (0, 4)$

ES

$$\frac{1}{x+1} > \frac{1}{4}$$

$$\frac{1}{x+1} - \frac{1}{4} > 0$$

$$\frac{4-(x+1)}{4(x+1)} > 0$$

$$\frac{4-x-1}{4(x+1)} > 0$$

$$\frac{3-x}{4(x+1)} > 0$$

f

g

h

$f(x) > 0$

$3 - x > 0$

$-x > -3$

$\boxed{x < 3}$

$4 > 0$

$\mathbb{R}$

$x + 1 > 0$

$x > -1$

$-1 < x < 3$

$S = (-1, 3)$

$$\frac{x^2-2}{2x-3} \geq 1$$

$$\downarrow$$

$$\frac{x^2-2}{2x-3} - 1 \geq 0$$

$$\frac{x^2-2-2x+3}{2x-3} \geq 0$$

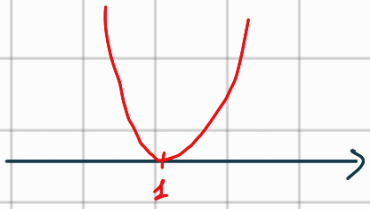
$$\frac{x^2-2x+1}{2x-3} \geq 0$$

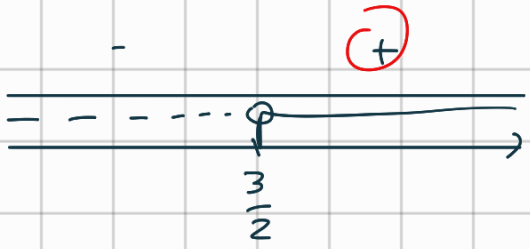
$2x-3 > 0$

$x > \frac{3}{2}$

$x^2-2x+1 \geq 0$

$\Delta = 4-4 = 0$





$(x-1)^2 \geq 0$

$\mathbb{R}$

$\left(\frac{3}{2}, +\infty\right)$

- $\frac{x^2+2x-5}{x^2-6x+8} \leq 0$

- $\frac{3}{x-2} < \frac{2x}{3+x}$