Lezione 7

07/11/23

$$f(x)=\frac{\sqrt{1-x^2}}{3x} \qquad f: D \longrightarrow \mathbb{R} \qquad D \subseteq \mathbb{R}$$

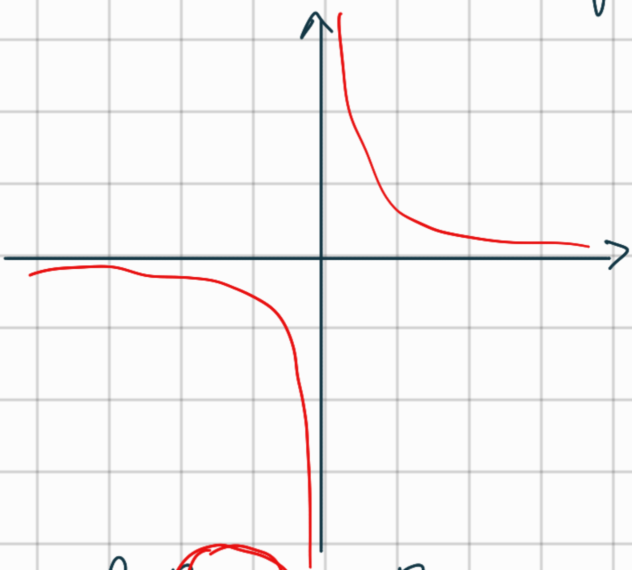
Vogliamo disegnarla → STUDIO DI FUNZIONE

Il primo passo è sapere qual è il dominio della funzione cioè per quali valori di x la funzione ha senso

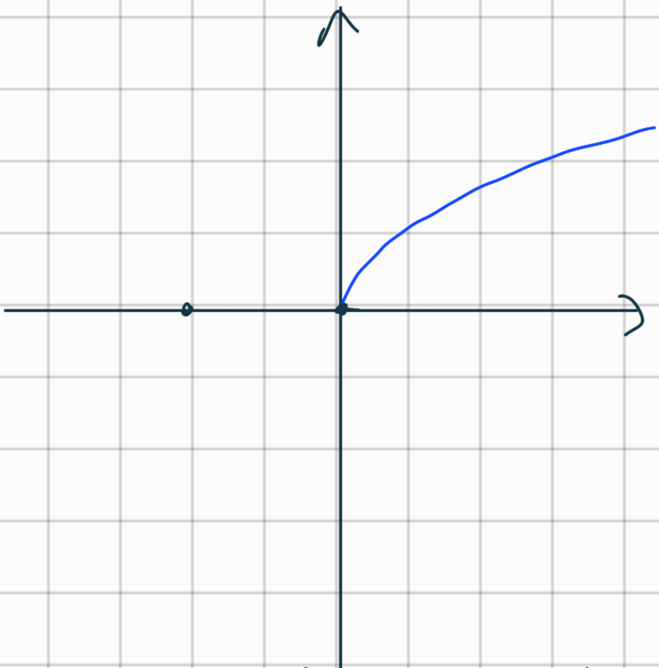
CONDIZIONI DI ESISTENZA

Regola (1) Non si può dividere per 0

Regola (2) Non si fa la radice quadrata di un numero negativo



$$f: \mathbb{R}\setminus\{0\} \longrightarrow \mathbb{R}\setminus\{0\}$$
$$x \longmapsto \frac{1}{x}$$



$$\sqrt{\cdot}: [0,+\infty) \longrightarrow [0,+\infty)$$
$$x \longmapsto \sqrt{x}$$

$$① \Rightarrow \begin{cases} 3x \neq 0 \\ 1 - x^2 \geq 0 \end{cases} \Leftarrow ② \rightarrow \begin{cases} x \neq 0 \\ x \in [-1, 1] \end{cases}$$

$-x^2 + 1 \geq 0$ DISCORDI

EQ. ASSOCIATA

$-x^2 + 1 = 0$

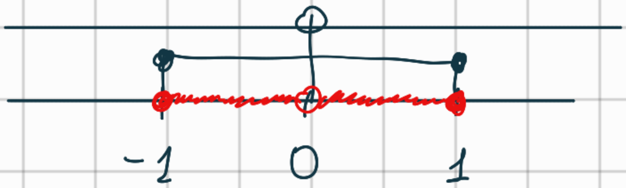
$\Delta = 4$

$$x_{1\,2} = \frac{\pm\sqrt{4}}{-2}$$

$$x_1 = \frac{2}{-2} = -1$$

$$x_2 = \frac{-2}{-2} = 1$$

$[-1, 1]$



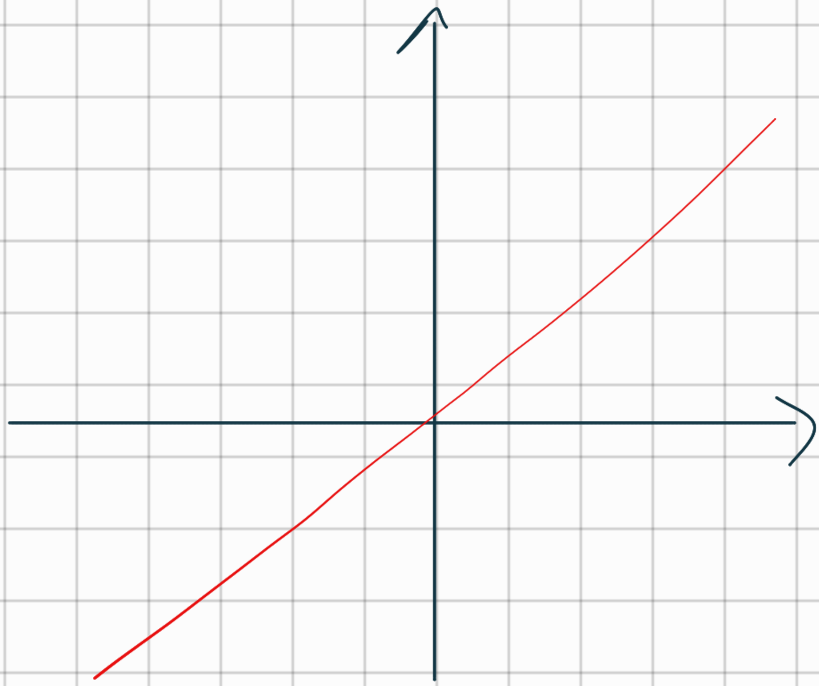
$D = [-1, 0) \cup (0, 1]$

$$f(x) = \frac{\sqrt{\pi + 2}}{3}\, x$$

$D = \mathbb{R}$

$\pi + 2 \geq 0 \quad \mathbb{R}$

$3 \neq 0 \quad \mathbb{R}$



$$f(x) = \left(\frac{x}{x+3}\right)^5 \qquad x \neq -3$$

C.E.

$$x + 3 \neq 0$$

$$x \neq -3 \qquad D = (-\infty, -3) \cup (-3, +\infty)$$

$$\sqrt{\frac{x^2 - 2x + 1}{x^2 - 5}}$$

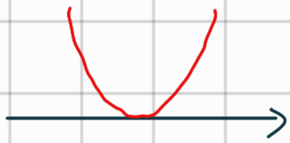
$$\begin{cases} x^2 - 5 \neq 0 \\ \boxed{\dfrac{x^2 - 2x + 1}{x^2 - 5} \geq 0} \end{cases} \rightarrow \begin{cases} x \neq \pm\sqrt{5} \\ (-\infty, -\sqrt{5}) \cup (\sqrt{5}, +\infty) \end{cases} \rightarrow (-\infty, -\sqrt{5}) \cup (\sqrt{5}, +\infty)$$

$$+x^2 - 2x + 1 \geq 0 \qquad\qquad x^2 - 5 > 0$$

MODO 1

$$\Delta = 4 - 4 = 0$$

$$x_1 = x_2 = \frac{2}{2} = 1$$



$\mathbb{R}$

MODO 2

$$(x-1)^2 \geq 0$$

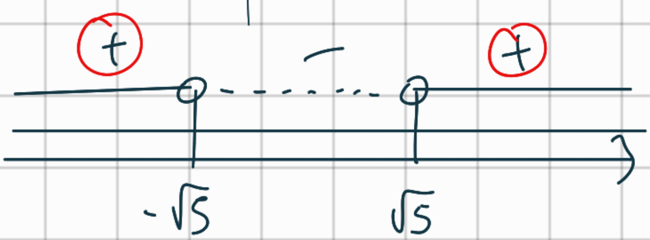
EQ. ASSOCIATA

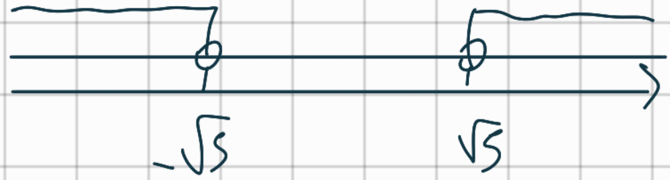
$$x^2 - 5 = 0$$

$$x^2 = 5$$

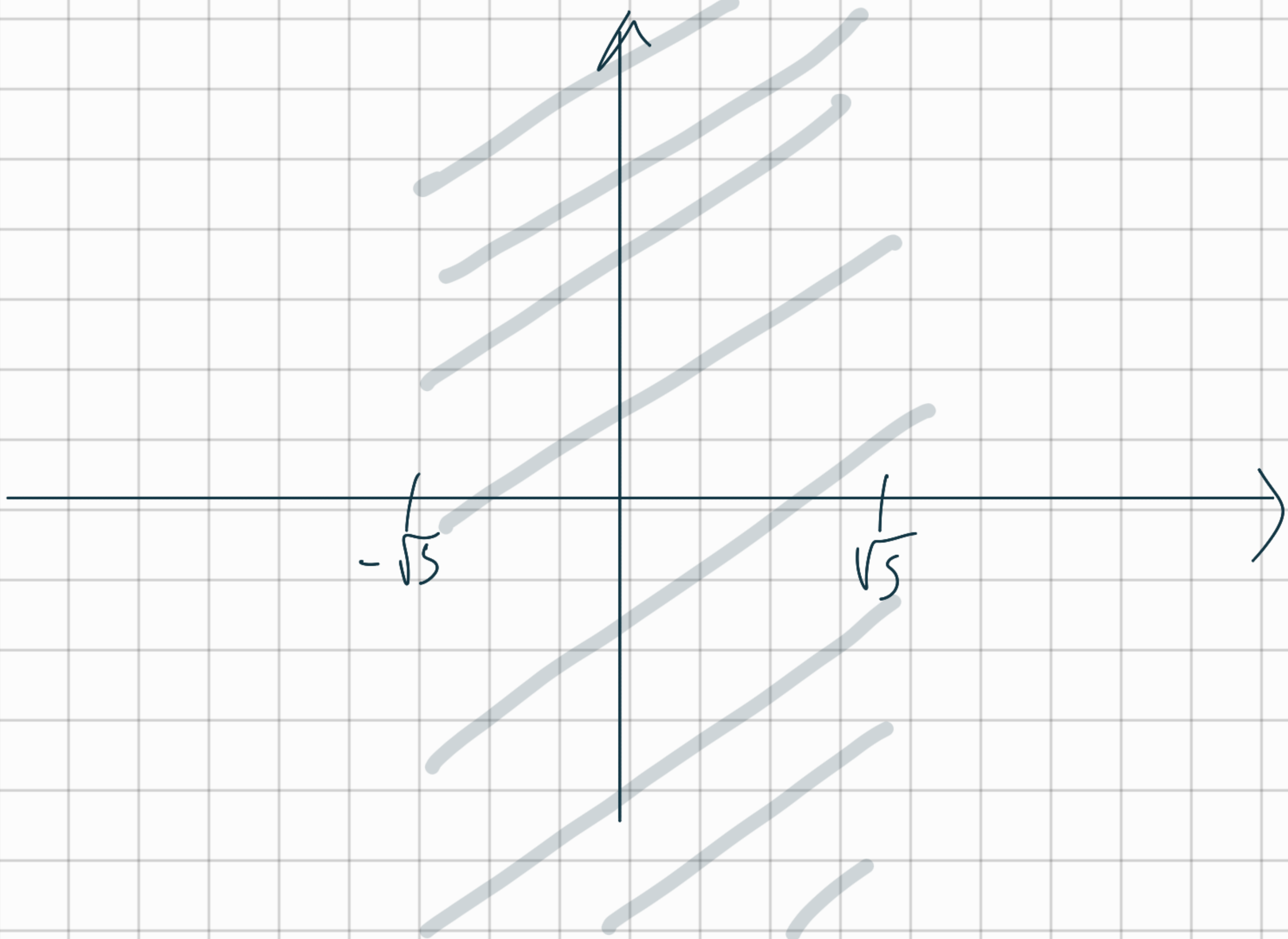
$$x_{1,2} = \pm\sqrt{5}$$

$$(-\infty, -\sqrt{5}) \cup (\sqrt{5}, +\infty)$$



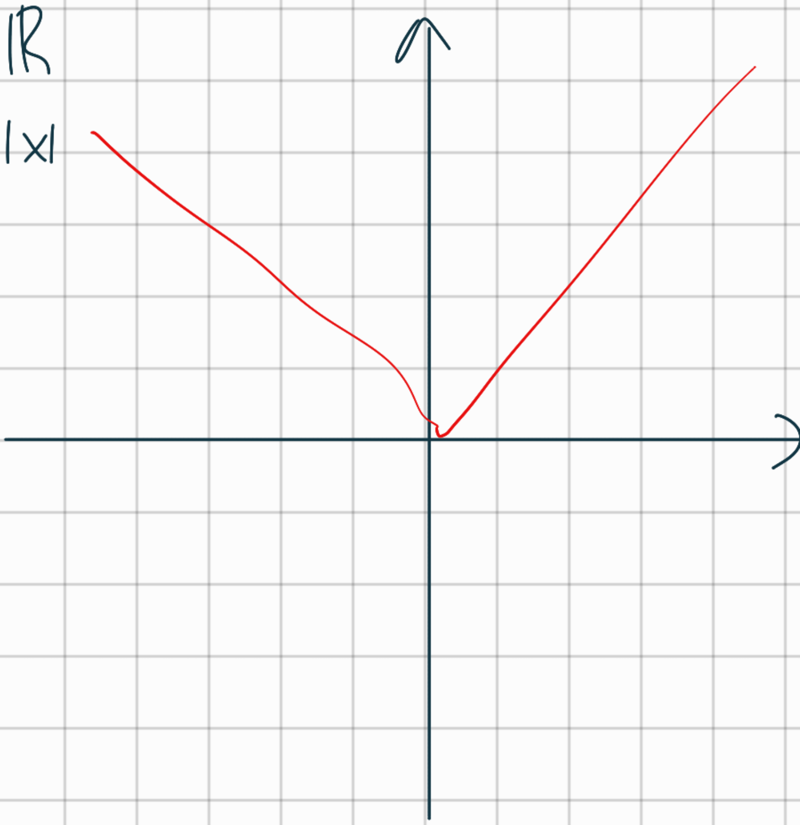


$(-\infty, -\sqrt{5}) \cup (\sqrt{5}, +\infty)$



NOTARE

$$|\cdot| : \mathbb{R} \to \mathbb{R}$$
$$x \mapsto |x|$$



$$f(x) = \frac{x}{|x+2|-3} \Rightarrow D = \mathbb{R} \setminus \{-5, 1\}$$

$$|x+2|-3 \neq 0$$
$$\Downarrow$$
$$|x+2| \neq 3$$

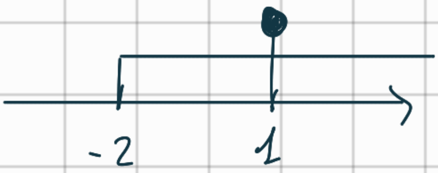
Cerchiamo i punti $|x+2| = 3$

(1) $\boxed{x+2=3 \wedge x+2 \geq 0}$

(2) $\boxed{-x-2=3 \wedge x+2 < 0}$

(1)

$$\begin{cases} x \geq -2 \\ x = 1 \end{cases}$$



$$\begin{cases} x < -2 \\ -x = 5 \end{cases} \rightarrow \begin{cases} x < -2 \\ x = -5 \end{cases}$$

$\{1\}$ $\{-5\}$

VERIFICA

$|x+2|-3$

$|1+2|-3$

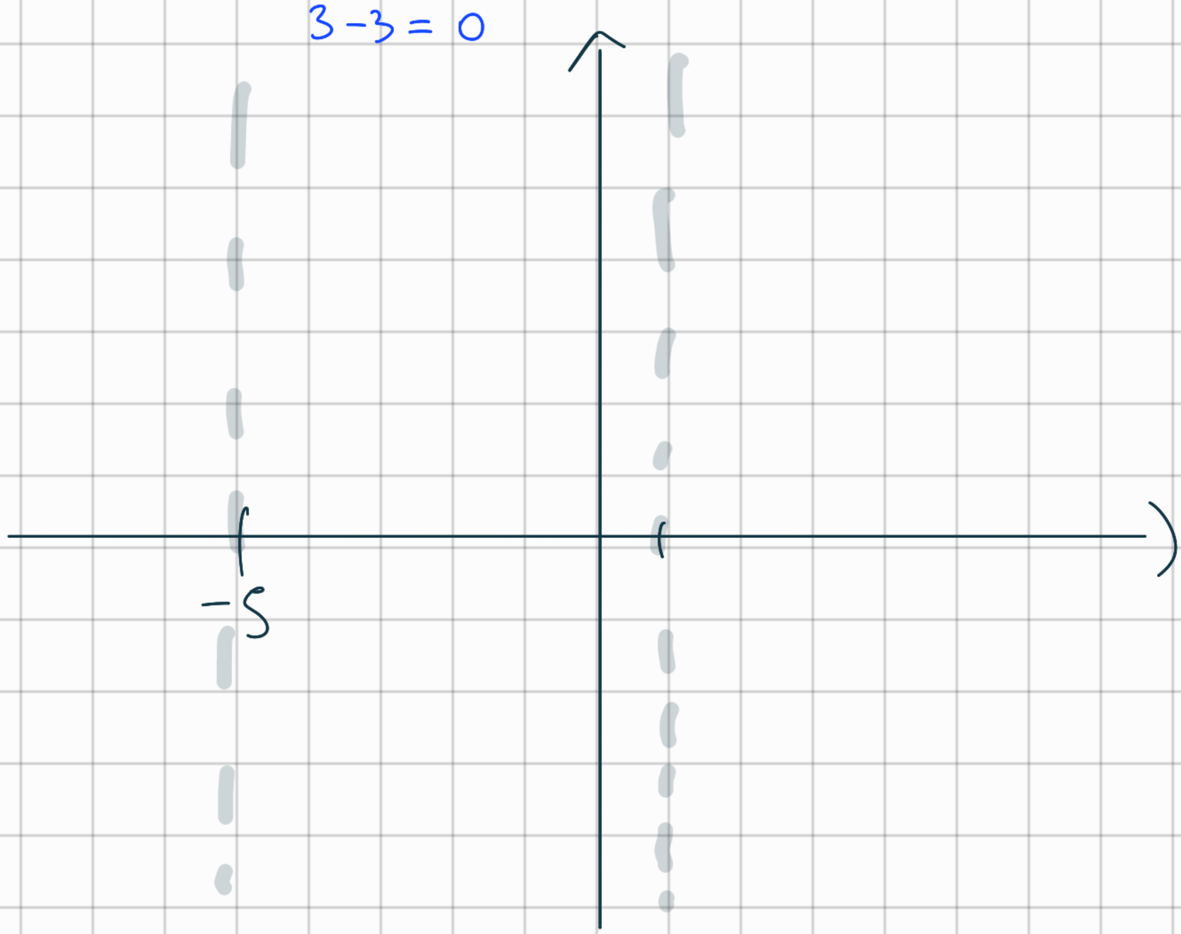
$|3|-3$

$3-3=0$

$|-5+2|-3$

$|-3|-3$

$3-3=0$

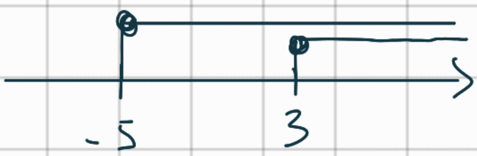


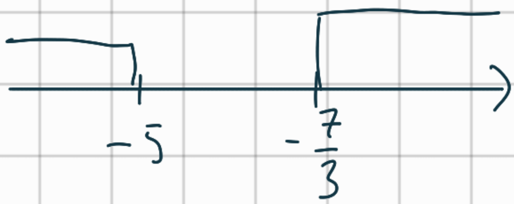
$$|x+5| \leq 2x+2$$

$$\begin{cases} x+5 \leq 2x+2 \\ x+5 \geq 0 \end{cases} \vee \begin{cases} -x-5 \leq 2x+2 \\ x+5 < 0 \end{cases}$$

$$\begin{cases} -x \leq -3 \\ x \geq -5 \end{cases} \qquad \begin{cases} -3x \leq 7 \\ x < -5 \end{cases}$$

$$\begin{cases} x \geq 3 \\ x \geq -5 \end{cases} \qquad \begin{cases} x \geq -\frac{7}{3} \\ x < -5 \end{cases}$$





$[3, +\infty) \quad \cup \quad \varnothing$

$$-\frac{5 \cdot 3}{3} = -\frac{15}{3}$$

$[3, +\infty)$

Definizione

$f(A) = \{f(x) : x \in A\}$

Sia $f : A \to B$ $\quad g : C \to D$ $\quad$ con $f(A) \subseteq C$

$g \circ f$ è la funzione

$$\begin{array}{ccccc} A & \longrightarrow & C & \longrightarrow & D \\ x & \longmapsto & f(x) & \longmapsto & g(f(x)) \end{array}$$

ESEMPIO $\quad A = B = C = D = \mathbb{R}$

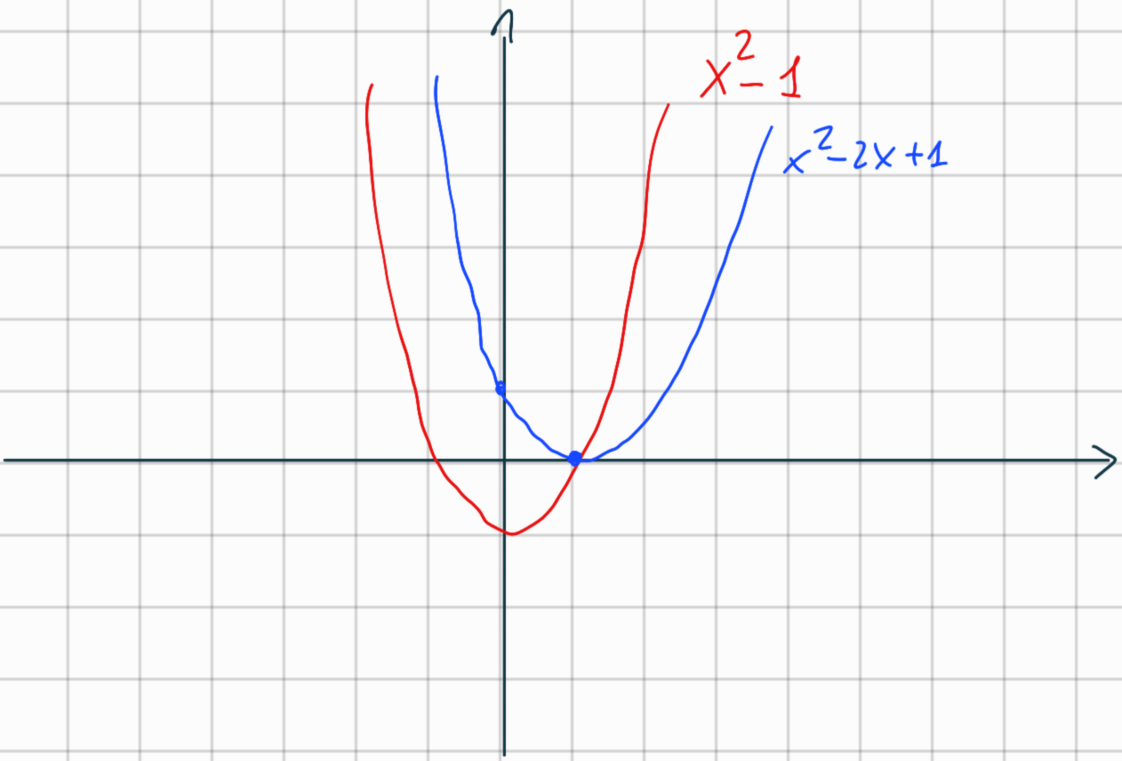
$f(x) = x^2$

$g(x) = x - 1$

$$g \circ f\,(x) = g(f(x)) = g(x^2) = x^2 - 1$$

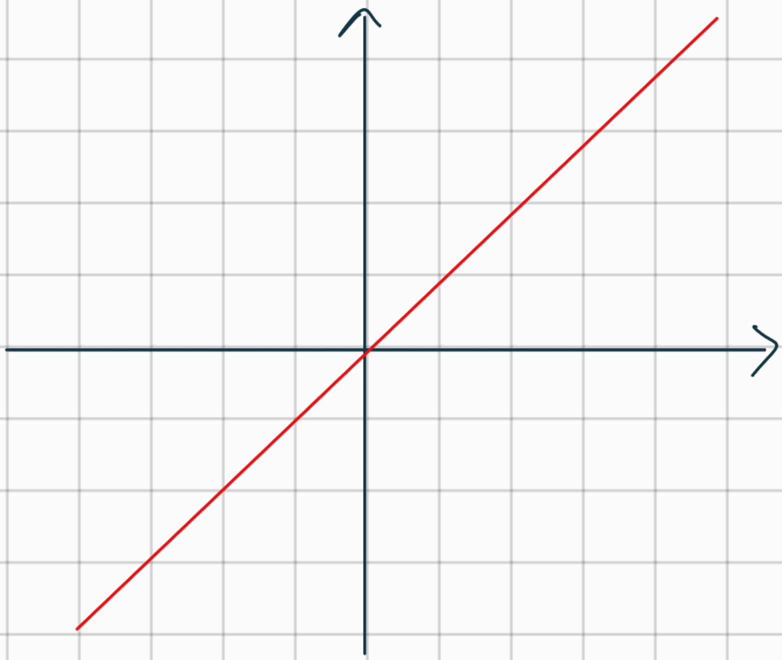
$$f \circ g\,(x) = f(x-1) = (x-1)^2 = x^2 - 2x + 1$$

OSSERVIAMO CHE la composizione NON È COMMUTATIVA



RICORDO

$$\begin{aligned} id_A : A &\to A \\ x &\mapsto x \end{aligned}$$



Una funzione BIETTIVA è INVERTIBILE $\quad f : A \to B$ BIETTIVA

La sua inversa è $f^{-1} : B \to A$ tale che

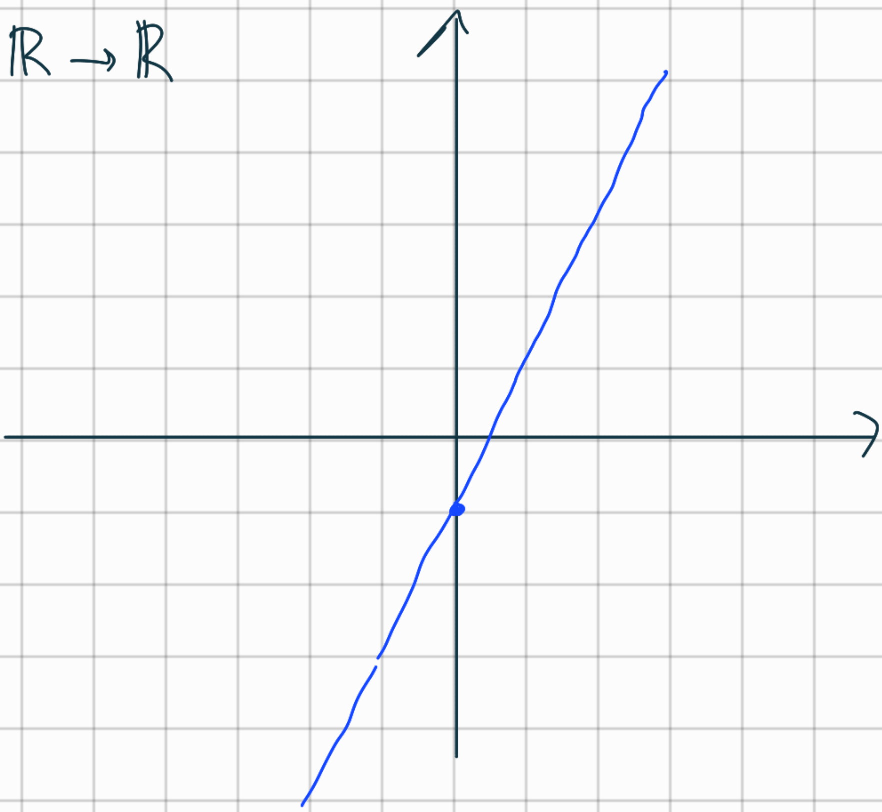
$$f^{-1} \circ f(x) = id_A \qquad id_A \quad A \xrightarrow{f} B \xrightarrow{f^{-1}} A$$

$$f \circ f^{-1}(x) = id_B \qquad id_B \quad B \xrightarrow{f^{-1}} A \xrightarrow{f} B$$

$f(x) = 2x - 1 \qquad f : \mathbb{R} \to \mathbb{R}$

È INIETTIVA ✓

SURIETTIVA ✓



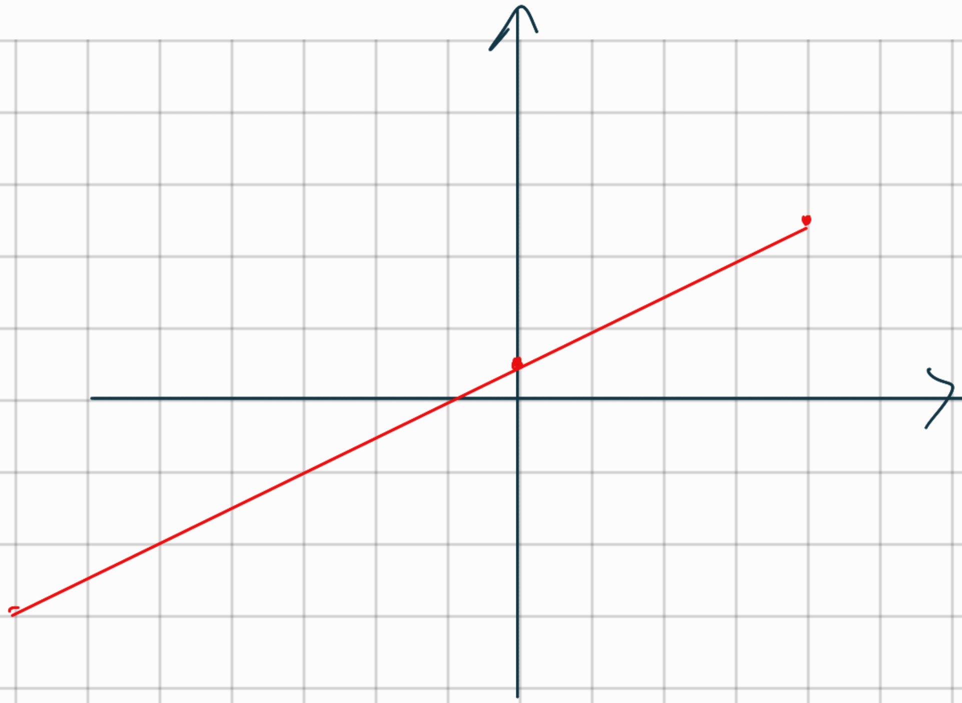
$$x = g(y) = \frac{y}{2} + \frac{1}{2}$$

$$y = 2x - 1$$

$$y + 1 = 2x$$

$$\boxed{\frac{y}{2} + \frac{1}{2} = x}$$

Disegno $g(x)$



Scrivere le funzioni $f(x) = \sqrt{x^2+1}$ come funzione composta

$$h(g(f(x))) = h(g(x^2)) = h(x^2+1) = \sqrt{x^2+1}$$

$$h: [0,+\infty) \longrightarrow [0,+\infty)$$
$$x \longmapsto \sqrt{x}$$

$$\mathbb{R} \longrightarrow \mathbb{R}$$
$$g: \quad x \longmapsto x+1$$

$$\mathbb{R} \longrightarrow \mathbb{R}$$
$$f: \quad x \longmapsto x^2$$

$$x^2+1 \geq 0 \Rightarrow \forall x \in \mathbb{R}$$

ALTRO ESEMPIO $\sqrt{x^2-1}$

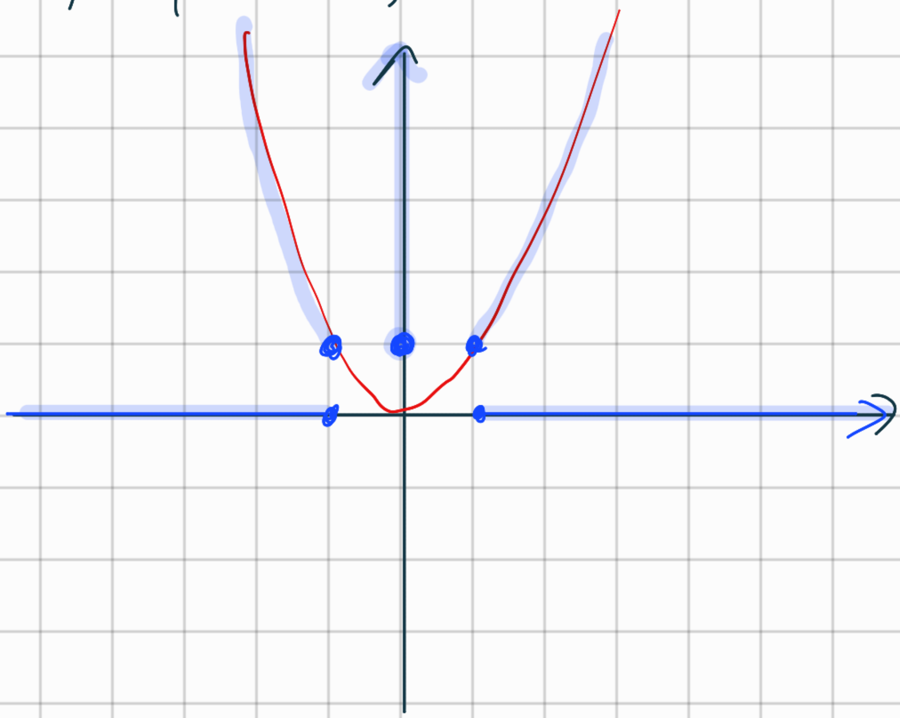
$$x^2 - 1 \geq 0$$

$$x^2 \geq 1$$

$$g : \mathbb{R} \longrightarrow \mathbb{R}$$
$$x \longmapsto x-1$$

$$\underbrace{x \in (-\infty, 1] \cup [1, +\infty)}= D \qquad h(g(f(x)))$$

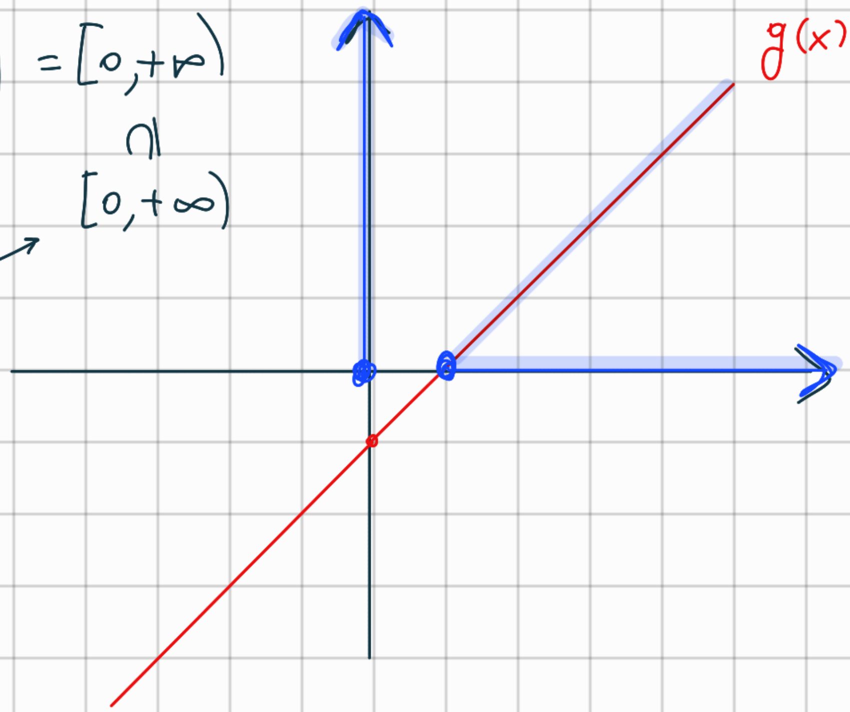
$$f(D) = \{f(x) : x \in D\} = \{x^2 : x \in D\} = [1, +\infty)$$



$$g(f(x)) = g([1, +\infty)) = [0, +\infty)$$
$$\cap$$
$$[0, +\infty)$$

è il dominio delle funzione h

ok

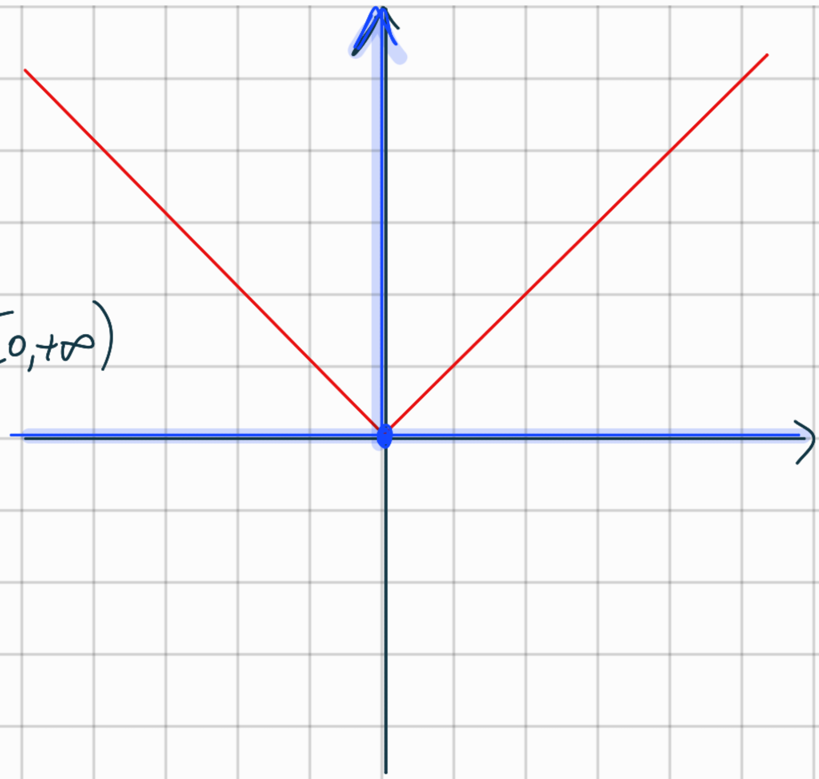


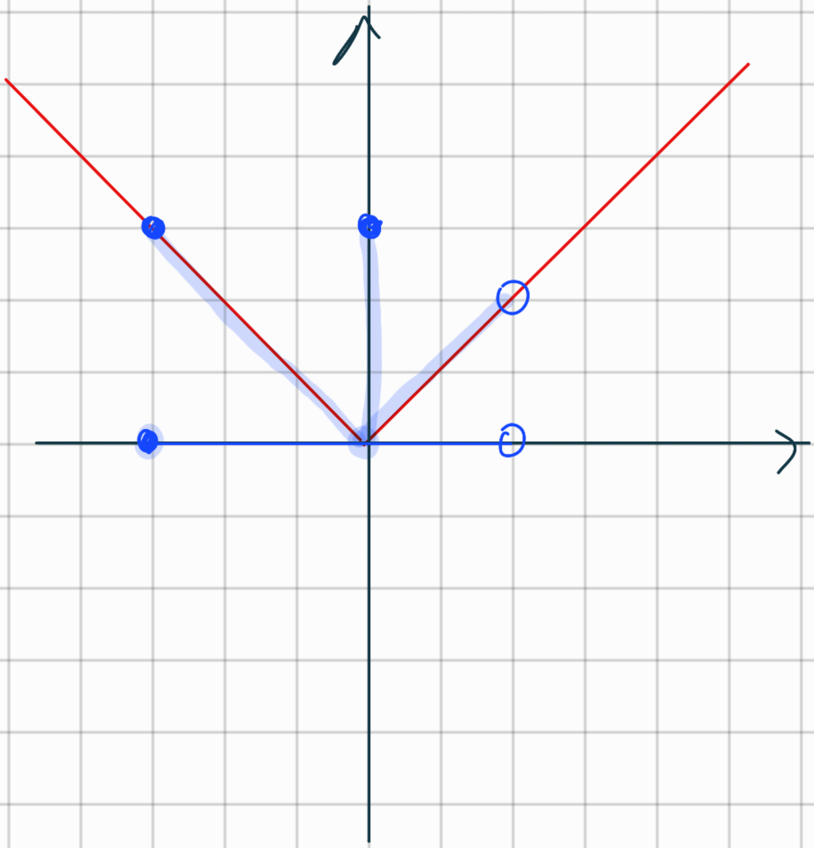
$f(x) = |x|$

$f : \mathbb{R} \to \mathbb{R}$

$f(\mathbb{R}) = \{ |x| \in \mathbb{R} : x \in \mathbb{R} \} = [0, +\infty)$

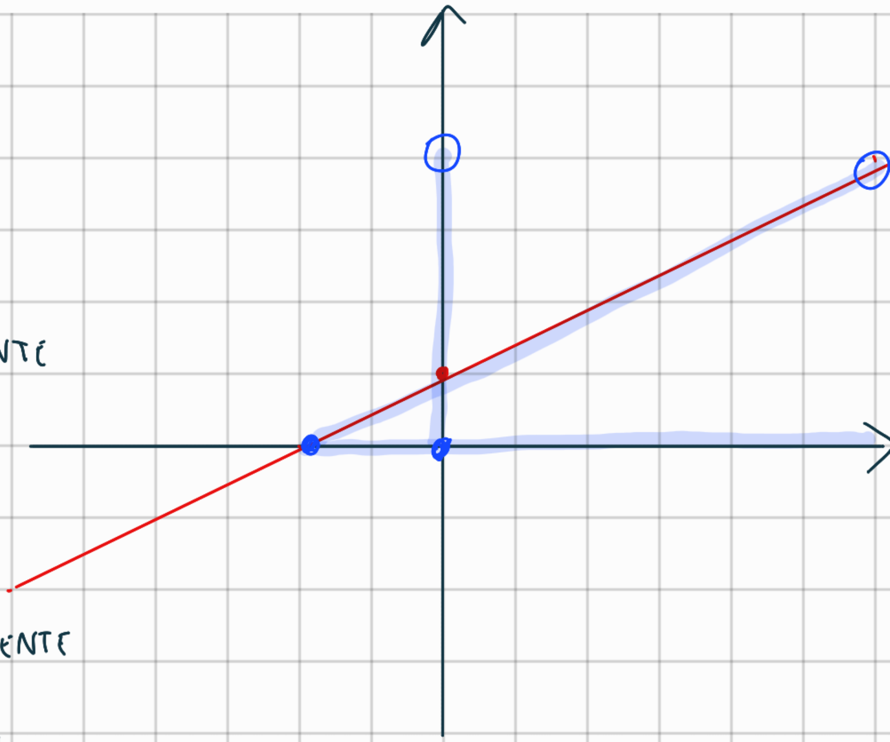
$f([-3, 2)) = [0, 3]$





$$f(x) = \frac{1}{2}x + 1$$

$$f([-2,6)) = [0,4)$$



SE f è monotone CRESCENTE

$$f([a,b)) = [f(a), f(b))$$

SE f è monotone DECRESCENTE

$$f([a,b)) = (f(b), f(a)]$$

$$f(-2) = \frac{1}{2}(-2) + 1 = -1 + 1 = 0$$

$$f(6) = \frac{1}{2}(6) + 1 = 3 + 1 = 4$$

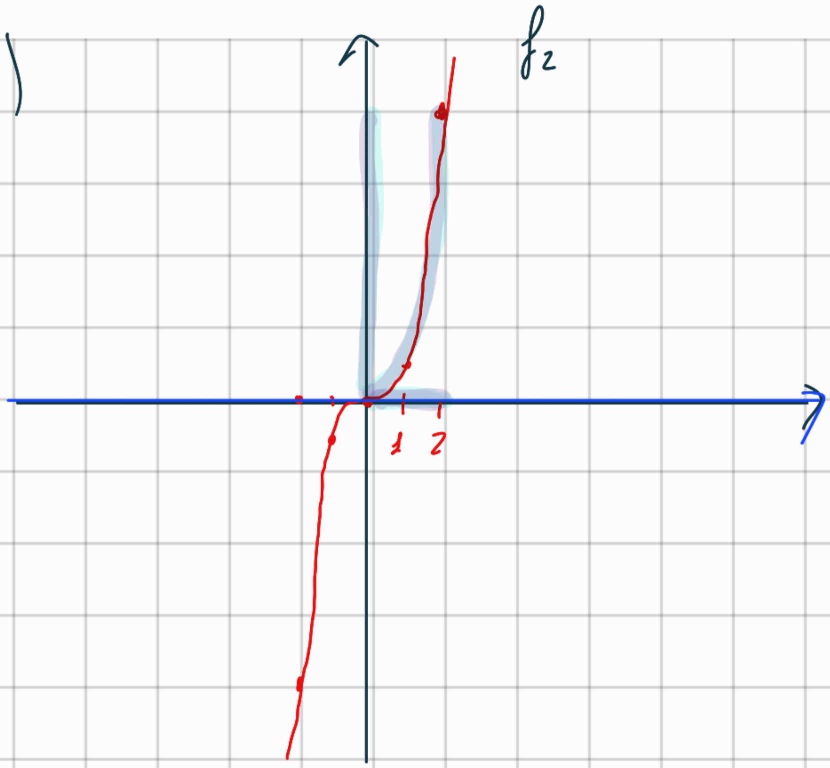
$$f(x) = -\frac{x^3}{3}$$ DIRE QUALI SONO GLI INSIEMI $f(\mathbb{R})$ e $f([0,2])$

$$= -\frac{1}{3}x^3 \qquad f_1(x) = -\frac{1}{3}x$$

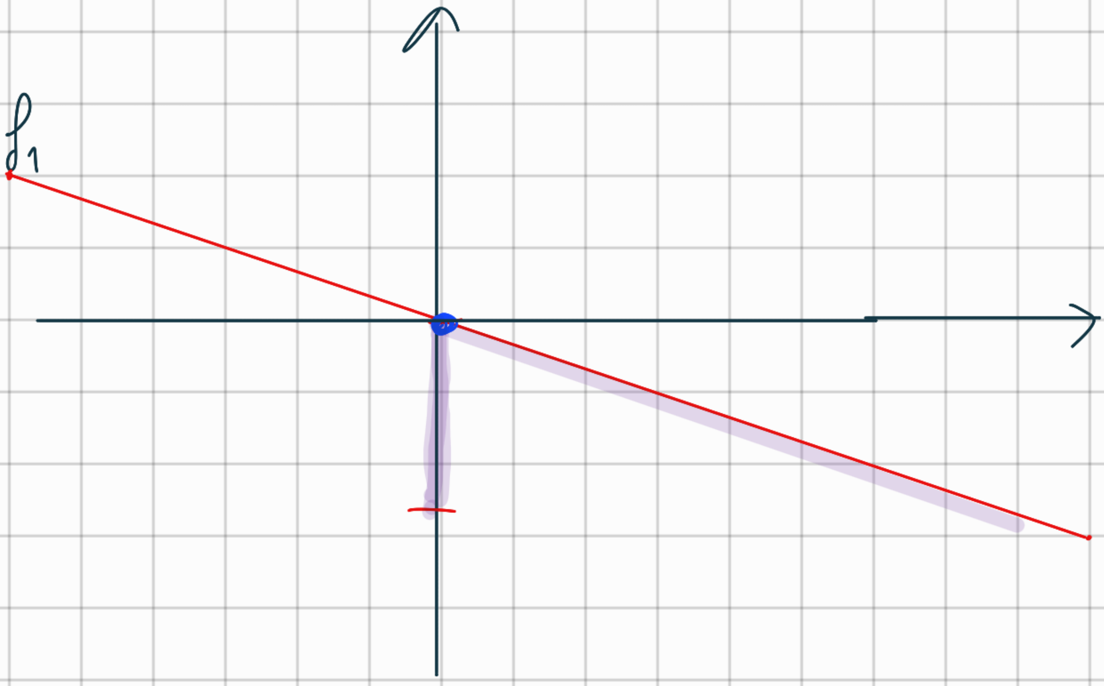
$$= f_1(f_2(x)) \qquad f_2(x) = x^3$$

$$f(\mathbb{R}) \quad f_1(f_2(\mathbb{R})) = f_1(\mathbb{R}) = \mathbb{R}$$

$$f([0,2]) = f_1(f_2([0,2])) = f_1([0,8])$$
$$= \left(-\frac{8}{3}, 0\right]$$



$$f_1(8) = -\frac{1}{3}8 = -\frac{8}{3}$$



ESERCIZIO

Se g è la funzione $\sqrt{\cdot}$

Dire per quali valori di x ha senso $g \circ f$