Lezione 10 14/11/23

Risolvere eq.

$$2^x = \frac{1}{8}$$

$$\frac{1}{8} = \frac{1}{2^3} = 2^{-3}$$

1° modo

Per def. di logaritmo $x = \log_2\left(\frac{1}{8}\right) = \log_2(2^{-3}) = -3$

(oppure $= -3 \log_2(2) = -3 \cdot 1 = -3$)

2° modo

$$2^x = \frac{1}{8}$$

$$2^x = 2^{-3}$$

$\Downarrow$ ← PER L'INIETTIVITÀ della funzione 2^x

$$x = -3$$

$$10^x = \frac{1}{3}$$

$$x = \log_{10}\left(\frac{1}{3}\right) = \log_{10}(3^{-1}) = -\log_{10}(3)$$

- $(2\sqrt{3})^x = 144$ $\qquad 144 = 12^2$

$(12^{\frac{1}{2}})^x = 144$ $\qquad 2\sqrt{3} = \sqrt{4}\cdot\sqrt{3} = \sqrt{12} = 12^{\frac{1}{2}}$

$12^{\frac{1}{2}x} = 144 \longrightarrow$ modo 2

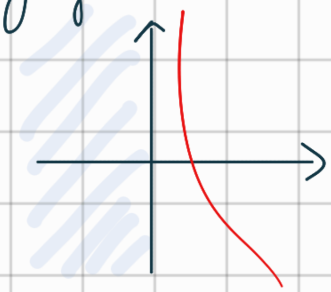
$12^{\frac{1}{2}x} = 12^2 \overset{\text{INIETTIVITÀ}}{\Longrightarrow} \frac{1}{2}x = 2 \rightarrow x = 4$

$$\frac{1}{2}x = \log_{12}(12^2)$$

$$x = 2 \cdot 2 = 4$$

Traccia il grafico della seguente curva

$$y = \log_{\frac{2}{3}} x$$



$$0 < \frac{2}{3} < 1$$

$$\log_{\frac{2}{3}}(1) = 0$$

$$\log_{\frac{2}{3}}\left(\frac{2}{3}\right) = 1$$

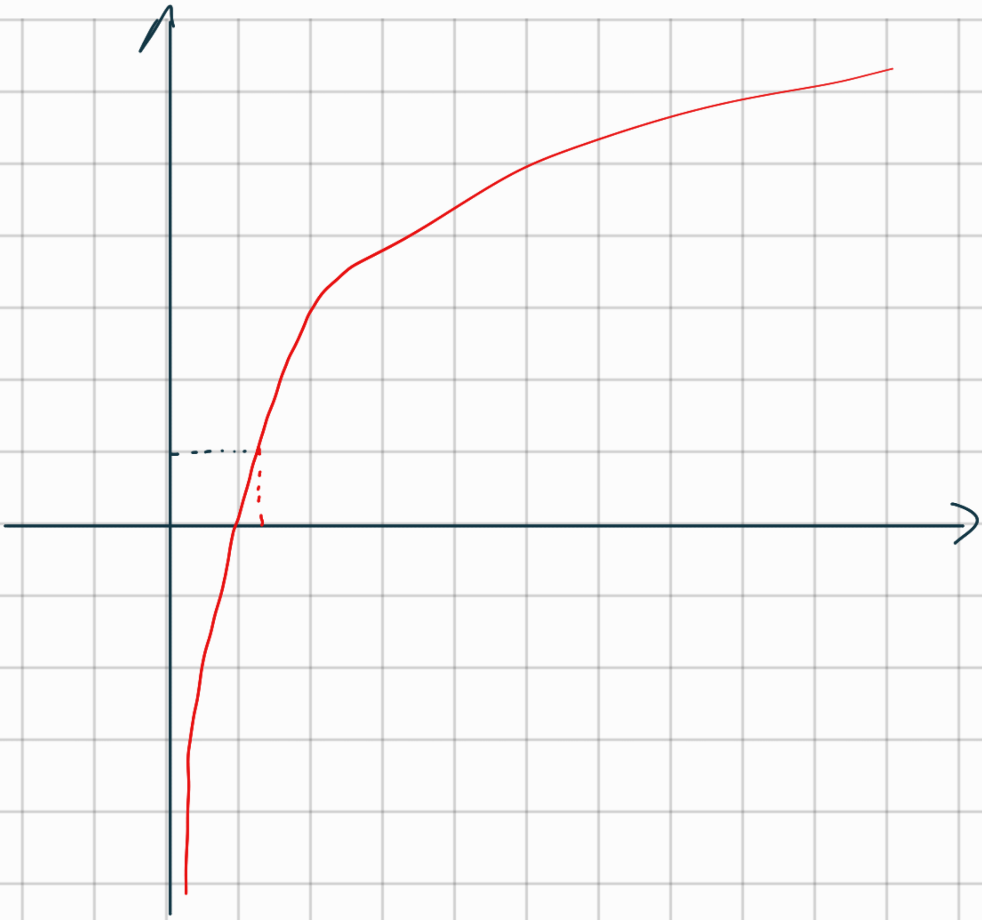
$$\log_{\frac{2}{3}}\left(\left(\frac{2}{3}\right)^2\right) = 2$$

$$\log_{\frac{2}{3}}\left(\frac{3}{2}\right) = -1$$

$$\log_{\frac{2}{3}}\left(\frac{9}{4}\right) = -2$$

PROSEGUIRE PER CASA

$\log_{\frac{4}{3}}(x)$



- $$\frac{1}{|e^x-1|}=1 \quad \begin{cases} \dfrac{1}{e^x-1}=1 & \text{se } x>0 \quad \text{se } e^x-1\geq 0 \\[2ex] \dfrac{1}{-e^x+1}=1 & \text{se } x<0 \quad \text{se } e^x-1<0 \end{cases}$$

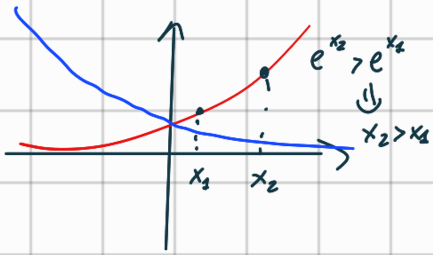
$e^x-1\geq 0$

$e^x\geq 1$

$e^x\geq e^0$ SICCOME

$\Downarrow \Leftarrow e>1$ CONSERVO IL VERSO

$x\geq 0$



C.E.

$|e^x-1|\neq 0$

$e^x-1\neq 0$

$e^x\neq 1$

$x\neq 0$

CASO $x>0$

$$\frac{1}{e^x-1}=1$$

$x\neq 0$

$$\rightarrow \frac{1}{\cancel{e^x-1}}\cancel{(e^x-1)}=1\cdot(e^x-1)$$

$$1=e^x-1$$

$$e^x=2$$

$$\downarrow$$

$$x=\ln 2$$

SICCOME $\ln 2>0$

ACCETTO LA SOLUZIONE ✓

CASO $x<0$

$$\frac{1}{-e^x+1}=1$$

$$1=-e^x+1$$

$$0=-e^x$$

$$e^x=0$$

$$\downarrow$$

NO SOLUZIONI

$$S=\{\ln 2\}$$

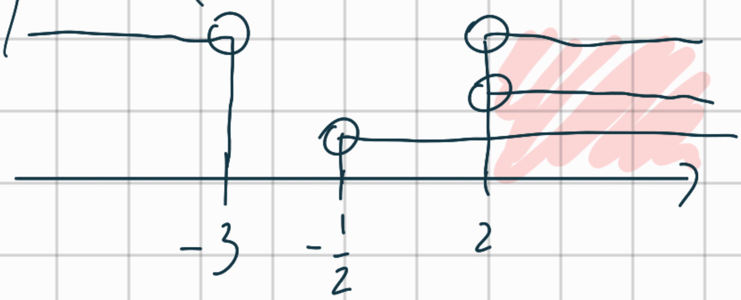
$$\log_a(x^2+x-6)=\log_a(x-2)+\log_a(2x+1)$$

C.E.

$$\begin{cases} x^2+x-6>0 \\ x-2>0 \\ 2x+1>0 \end{cases} \qquad \begin{cases} x\in(-\infty,-3)\cup(2,+\infty) \\ x\in(2,+\infty) \\ x\in\left(-\frac{1}{2},+\infty\right) \end{cases}$$

$$\Delta=1+24=25$$

$$x_{1,2}=\frac{-1\pm5}{2} \begin{cases} \nearrow x_1=-3 \\ \searrow x_2=2 \end{cases}$$



C.E. $x>2$

$$\log_a(x^2+x-6) = \log_a(x-2) + \log_a(2x+1)$$

$$\log_a\big((x-2)(x+3)\big) - \log_a(x-2) - \log_a(2x+1) = 0$$

$$\log_a\left(\frac{\cancel{(x-2)}(x+3)}{\cancel{(x-2)}(2x+1)}\right) = 0$$

$$\Updownarrow$$

C.E.

$x \neq -\frac{1}{2}$ perché $x > 2$

$$\frac{x+3}{2x+1} = 1$$

$$x+3 = 2x+1$$

$$2 = x$$

$$x = 2$$

NON ACCETTIAMO LA SOLUZIONE $\qquad S = \emptyset$

$\ln^2(x) = (\ln(x))^2$

$$\ln^2(x) + \ln(x) - 6 = 0 \qquad \text{C.E. } x > 0$$

CHIAMO $t = \ln(x)$

$$t^2 + t - 6 = 0$$

$$\Delta = 1 + 24 = 25$$

$$t_{1,2} = \frac{-1 \pm \sqrt{25}}{2}$$

$$t_1 = \frac{-1-5}{2} = \frac{-6}{2} = -3$$

$$t_2 = \frac{-1+5}{2} = \frac{4}{2} = 2$$

$\ln(x) = -3$ oppure $\ln(x) = 2$

$$e^{-3} = x \qquad\qquad x = e^2$$

$$x = e^{-3} = \frac{1}{e^3}$$

$$S = \{e^2, e^{-3}\}$$

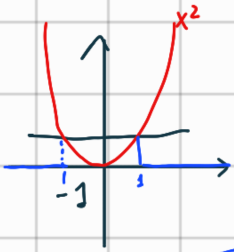
- $\ln \frac{x^2-1}{x} = \ln 2$

$\ln$ e' INIETTIVA

C.E. $\frac{x^2-1}{x} > 0$

$$\frac{x^2-1}{x} = 2$$

ABBIATO GIA' IMPOSTO CHE $x \neq 0$

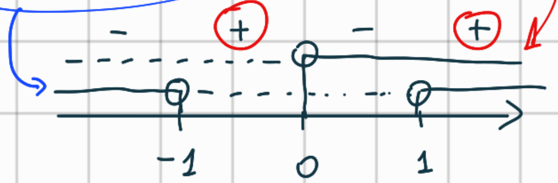


$x^2-1>0$

$x^2>1$

$x<-1 \vee x>1$

$x>0$



$x^2-1 = 2x$

$x^2-2x-1=0$ $\qquad \sqrt{8} = \sqrt{4 \cdot 2} = \sqrt{4} \cdot \sqrt{2} = 2\sqrt{2}$

$\Delta = 4+4 = 8$

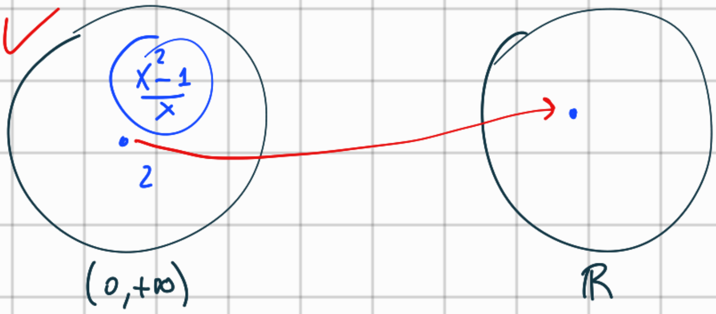
$$x_{12} = \frac{2 \pm 2\sqrt{2}}{2}$$

$x_1 = \frac{\cancel{2}(1-\sqrt{2})}{\cancel{2}} = 1-\sqrt{2} \in (-1,0)$ ✓

$x_2 = 1+\sqrt{2} \in (1,+\infty)$ ✓

C.E. $-1<x<0 \vee x>1$

$x \in (-1,0) \cup (1,+\infty)$



$\sqrt{1} < \sqrt{2}$

$1 < \sqrt{2}$

$1-1 < \sqrt{2}-1$

$0 < \sqrt{2}-1$

$0 > 1-\sqrt{2}$

$\boxed{1-\sqrt{2} < 0}$

$\sqrt{2} < \sqrt{4}$

$\sqrt{2} < 2$

$\sqrt{2}-1 < 2-1$

$\sqrt{2}-1 < 1$

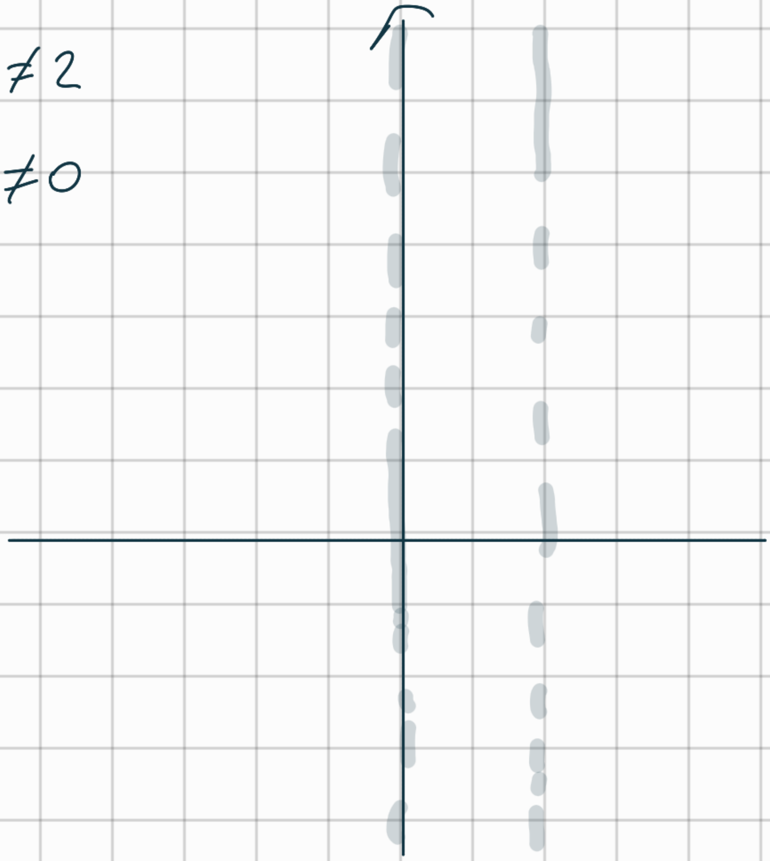
$\boxed{1-\sqrt{2} > -1}$

$S = \{1 \pm \sqrt{2}\}$

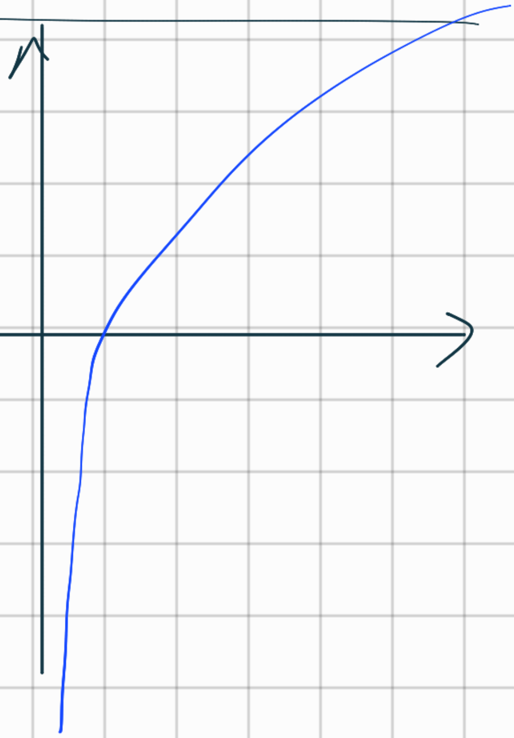
$$f(x) = \frac{\ln|x|}{x-2}$$

$$\begin{cases} x-2 \neq 0 \\ |x| > 0 \end{cases} \rightarrow \begin{cases} x \neq 2 \\ x \neq 0 \end{cases}$$

$$D = (-\infty, 0) \cup (0, 2) \cup (2, +\infty)$$



$$f(x) = \frac{\sqrt{x^2-4}}{\ln(3x-1)}$$



$$\begin{cases} x^2-4 \geq 0 \\ 3x-1 > 0 \\ \ln(3x-1) \neq 0 \end{cases} \rightarrow \begin{cases} x < -2 \vee x > 2 \\ x > \frac{1}{3} \\ 3x-1 \neq 1 \rightarrow x \neq \frac{2}{3} \end{cases}$$

$3x > 1$

$x > \frac{1}{3}$

$x^2 - 4 \geq 0$

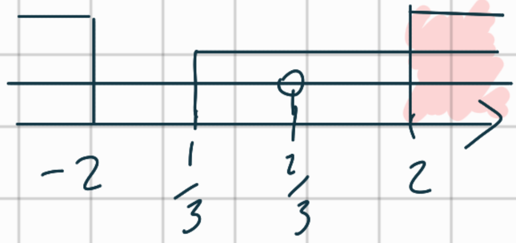
$\Delta = 16$

$x_{12} = \frac{\pm\sqrt{16}}{2}$ → $x_1 = -2$, $x_2 = 2$

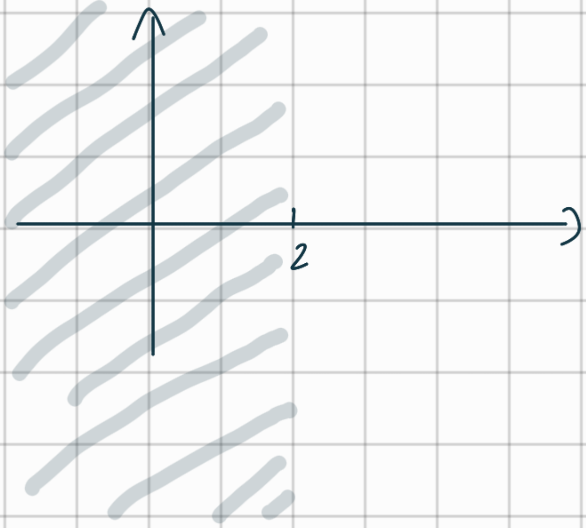
INTERVALLI ESTERNI

$x \leq -2 \vee x \geq 2$

Verifico dove valgono tutte
le condizioni contemporaneamente



$$D = [2, +\infty)$$



Il numero di batteri presenti in una coltura è 1000
Sappiamo che raddoppiano in due ore.
Quanti batteri ci sono dopo 3 ore e mezza?

$$N(t) = N_0 R^t$$

t : ore trascorse dall'inizio dell'esperimento.

$N_0 = 1000$

$$2 \cdot N(t) = N(t+2)$$

$$2 \cdot N_0 R^t = N_0 R^{t+2}$$

DIVIDO PER N_0

$$2R^t = R^{t+2}$$

$$2 = \frac{R^{t+2}}{R^t} = R^{\cancel{t}+2-\cancel{t}} = R^2$$

$$2 = R^2$$

$$R = \pm\sqrt{2}$$

ESCLUDO LA SOLUZIONE NEGATIVA PERCHÉ LA BASE DELL'ESPONENZIALE $(R = a > 0)$

$$\Downarrow$$

$$\boxed{R = \sqrt{2}}$$

Soluzione $3.5 = 3 + \frac{1}{2} = \frac{6+1}{2} = \frac{7}{2}$

$\simeq 3.364$

$$N\left(\frac{7}{2}\right) = \left(\sqrt{2}\right)^{7/2} \cdot 1000 = \left(2^{1/2}\right)^{7/2} \cdot 1000 = 1000 \sqrt[4]{2^7}$$

$$= 1000 \cdot (3.364) = 3364$$