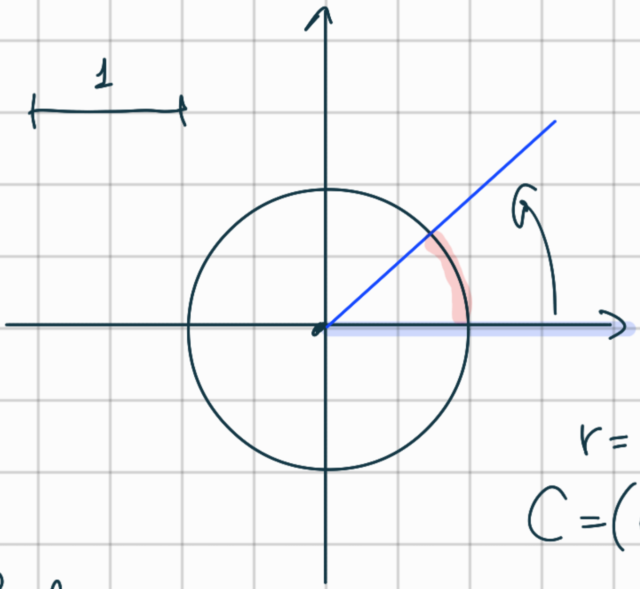
Lezione 11 — 17/11/23

FUNZIONI TRIGONOMETRICHE

CIRCONFERENZA GONIOMETRICA



$r = 1$

$C = (0,0)$

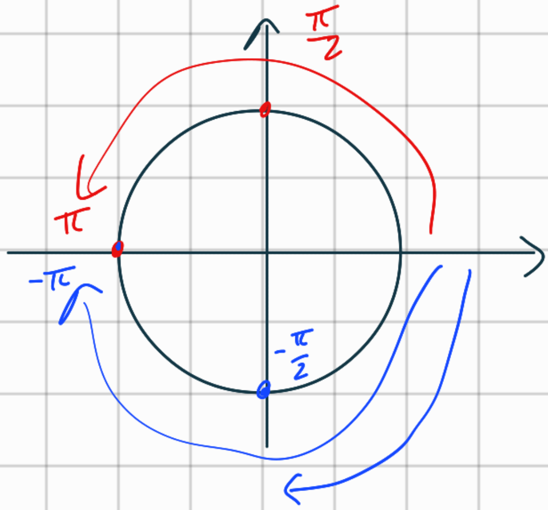
Gli angoli si misurano a partire dal semiasse positivo delle x

con il segno + in senso antiorario

La misura è in radianti cioè la lunghezza dell'arco di circonferenza percorso

L'angolo

GIRO	misura	$2\pi \overset{1}{\overset{\parallel}{r}} = 2\pi$
PIATTO	misura	π
RETTO	misura	$\frac{\pi}{2}$

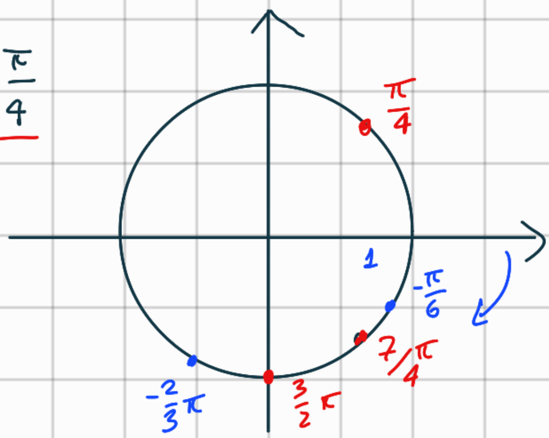


ESEMPIO

Disegnare sulla circ. goniometrica i seguenti angoli

$-\pi, \frac{3}{2}\pi, -\frac{2}{3}\pi, \frac{\pi}{4}, \frac{7}{4}\pi, -\frac{\pi}{6}, \frac{5}{6}\pi, -\frac{\pi}{4}$

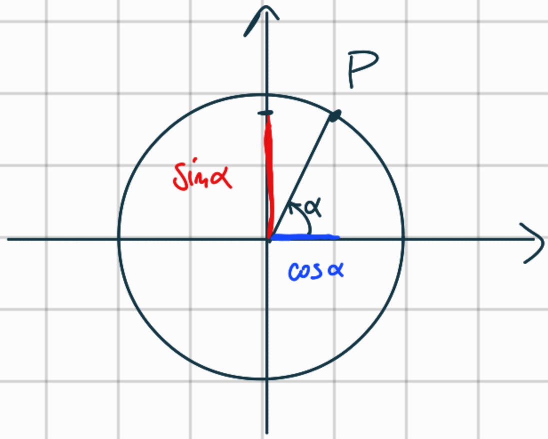
ESERCIZIO PER CASA



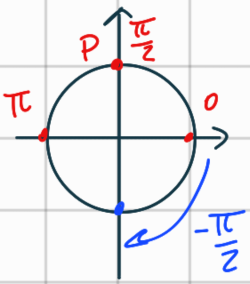
Definizione

Dato il punto $P = (x_P, y_P) \in \mathbb{R}^2$ sulla circ. goniometrica individuato dall'angolo α definiamo le funzioni $\sin(\alpha) := y_P$

$\cos(\alpha) := x_P$



$$\sin\left(\frac{\pi}{2}\right) = 1$$



	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3}{2}\pi$	2π
$\sin x$	-1	0	1	0	-1	0
$\cos x$	0	1	0	-1	0	1

$$\cos\left(\frac{\pi}{2}\right) = 0$$

$P \in \gamma$ circonferenza di centro $(0,0)$ e raggio 1

$$\gamma = \left\{ (x_P, y_P) : (x_P - 0)^2 + (y_P - 0)^2 = 1 \right\}$$

$$x_P^2 + y_P^2 = 1$$

EQUAZIONE FONDAMENTALE DELLA TRIGONOMETRIA

$$\boxed{\cos^2\alpha + \sin^2\alpha = 1}$$

VERIFICA $\cos^2\left(-\frac{\pi}{2}\right) + \sin^2\left(-\frac{\pi}{2}\right) = 0 + 1 = 1$

ATTENZIONE LA FORMULA VALE SE L'ANGOLO E' LO STESSO

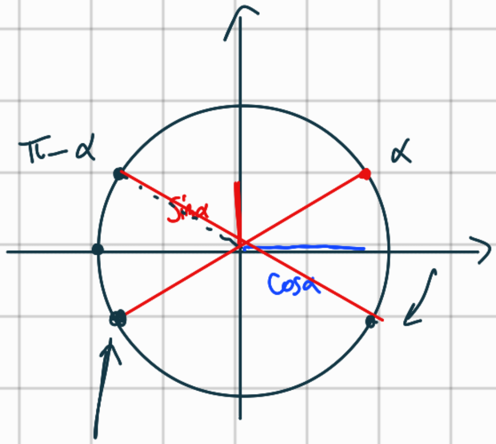
IN QUESTO CASO $-\frac{\pi}{2}$

$\sin(x)$ e $\cos(x)$ sono funzioni periodiche di periodo 2π

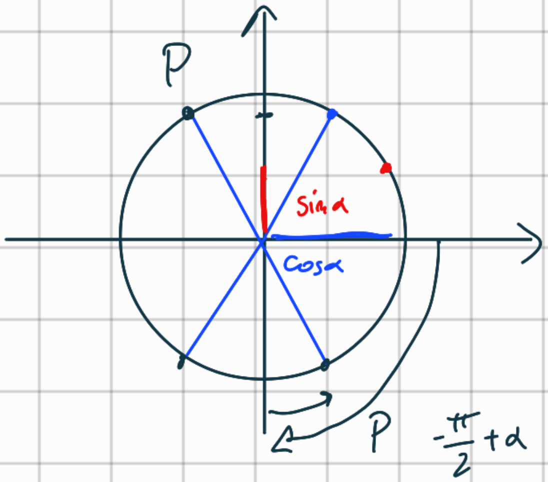
cioè $\longrightarrow$ $\sin(x) = \sin(x+2\pi) \quad \forall x$

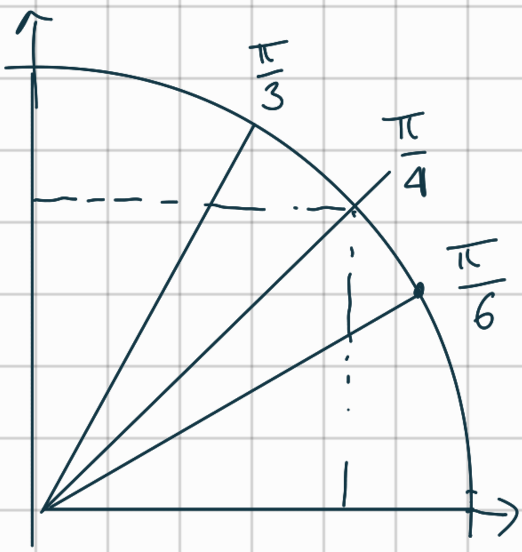
$\longrightarrow$ $\cos(x) = \cos(x+2\pi) \quad \forall x$

Ci sono degli angoli detti <u>ARCHI ASSOCIATI</u> che si possono ricavare a partire da un singolo angolo α



	sin	cos
$\alpha+2\pi$ | α	$\sin\alpha$	$\cos\alpha$
$\pi-\alpha$	$\sin\alpha$	$-\cos\alpha$
$\pi+\alpha$	$-\sin\alpha$	$-\cos\alpha$
$-\alpha$	$-\sin\alpha$	$\cos\alpha$
$\frac{\pi}{2}-\alpha$	$\cos\alpha$	$\sin\alpha$
$\frac{\pi}{2}+\alpha$	$\cos\alpha$	$-\sin\alpha$
$-\frac{\pi}{2}-\alpha$	$-\cos\alpha$	$-\sin\alpha$
$-\frac{\pi}{2}+\alpha$	$-\cos\alpha$	$\sin\alpha$



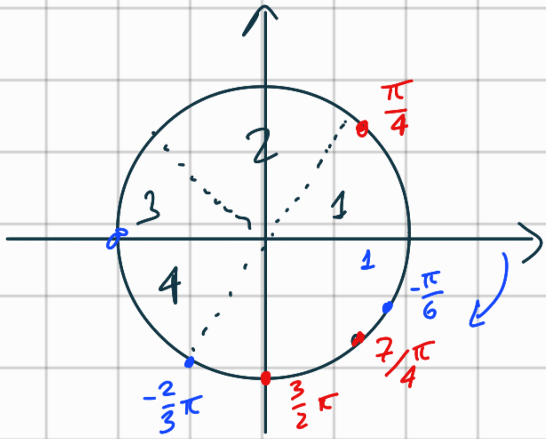


α	sin	cos
0	$0 = \frac{\sqrt{0}}{2}$	1
$\frac{\pi}{6}$	$\frac{1}{2} = \frac{\sqrt{1}}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{2}$	$1 = \frac{\sqrt{4}}{2}$	0

CERCHIAMO IL sin e cos DEGLI ANGOLI SCRITTI IN PRECEDENZA

$$-\pi,\ \frac{3}{2}\pi,\ -\frac{2}{3}\pi,\ \frac{\pi}{4},\ \frac{7}{4}\pi,\ -\frac{\pi}{6},\ \frac{5}{6}\pi,\ -\frac{\pi}{4}$$

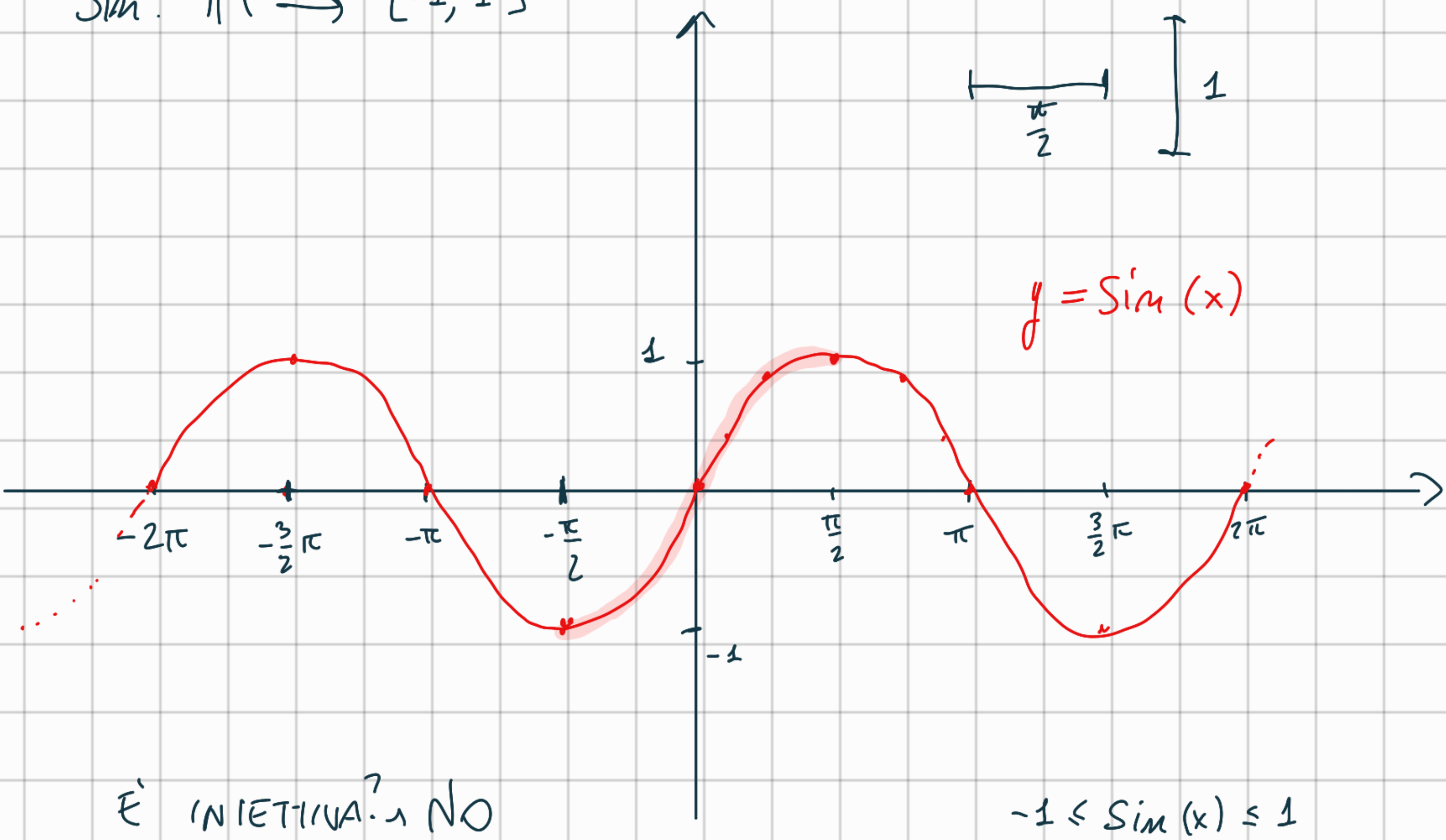
	sin	cos
$-\pi$	0	-1
$\frac{3}{2}\pi$	-1	0
$-\frac{2}{3}\pi$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$



← PROSEGUIRE PER CASA

LA FUNZIONE

$\sin : \mathbb{R} \to [-1, 1]$

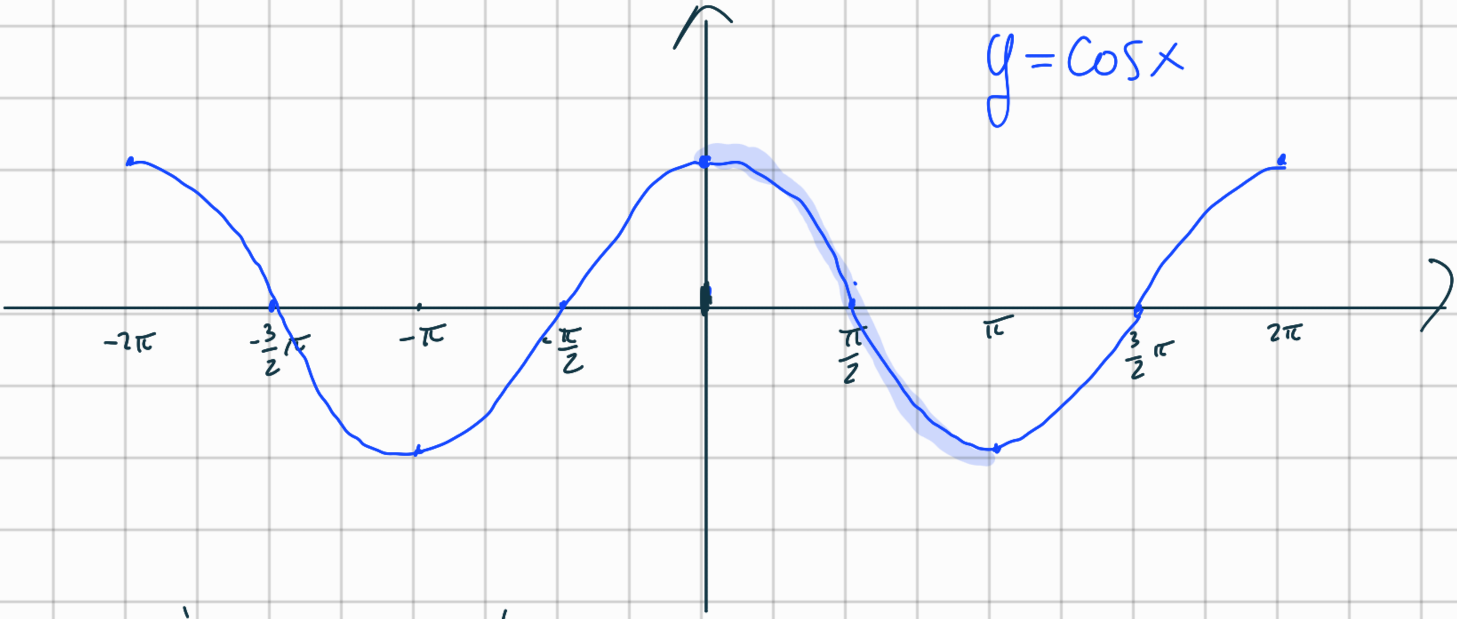


È INIETTIVA? → NO

È SURIETTIVA? → SÌ

$-1 \leq \sin(x) \leq 1$

LA FUNZIONE $\cos : \mathbb{R} \to [-1, 1]$



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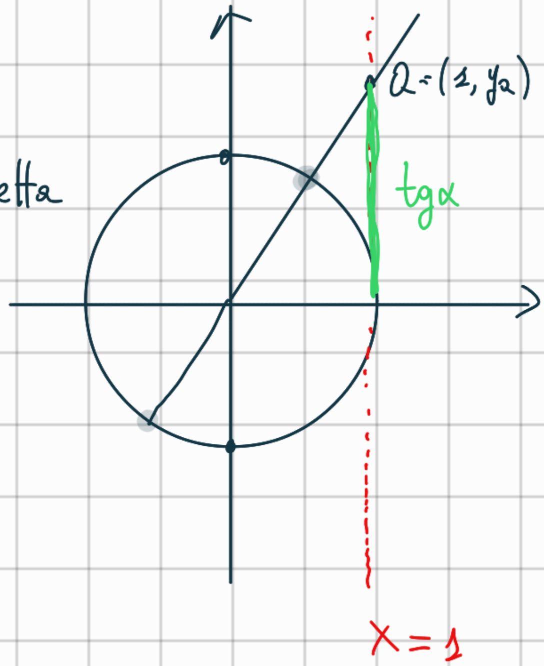
TANGENTE

$$tg(x) := \frac{\sin(x)}{\cos(x)}$$

$$tg : \mathbb{R} \setminus K \longrightarrow \mathbb{R}$$



$$tg(x) := y_Q$$

Q è il p.to di intersezione della semiretta che individua l'angolo e la retta $\boxed{x=1}$

$\boxed{\text{C.E. } \cos(x) \neq 0}$

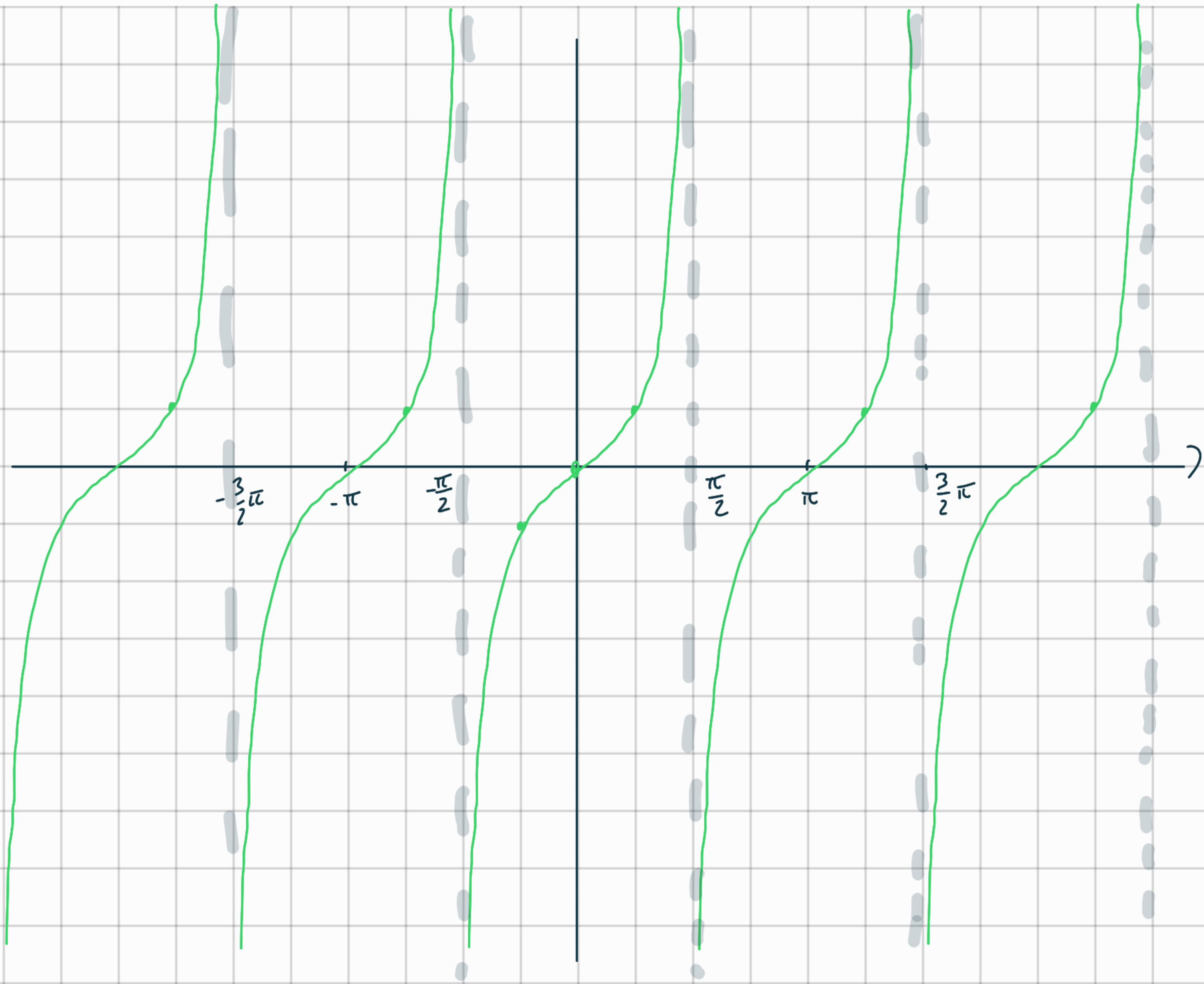
$$\Rightarrow K := \{x : x = \frac{\pi}{2} + k\pi, \ k \in \mathbb{Z}\}$$

$$\Downarrow$$

$$\cos x = 0 \Leftrightarrow x \in K$$

Regola ④ della C.E.

$$tg(f(x)) \Rightarrow \quad f(x) \neq \frac{\pi}{2} + k\pi \quad \forall k \in \mathbb{Z}$$



È INIETTIVA → NO

È SURIETTIVA → SÌ

tg è PERIODICA di periodo $\pi \Rightarrow tg(x+\pi) = tg(x)$

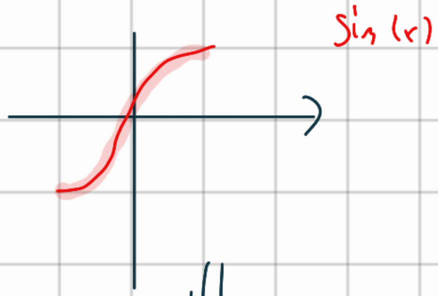
$$tg\,\frac{\pi}{4} = \frac{\sin\left(\frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{4}\right)} = \frac{\sqrt{2}/2}{\sqrt{2}/2} = \frac{\cancel{\sqrt{2}}}{\cancel{2}} \cdot \frac{\cancel{2}}{\cancel{\sqrt{2}}} = 1$$

$$tg\,-\frac{\pi}{4} = \frac{\sin\left(-\frac{\pi}{4}\right)}{\cos\left(-\frac{\pi}{4}\right)} = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = -1$$

Per poter calcolare le funzioni inverse dobbiamo restringere il dominio

$\sin : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \longrightarrow [-1, 1]$ così è INIETTIVA e SURIETTIVA

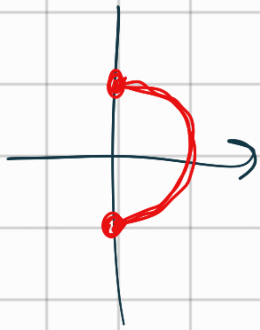
$\Downarrow$

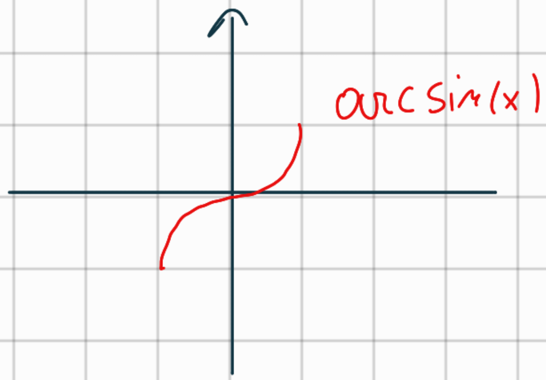


$\Downarrow$

$\arcsin : [-1, 1] \longrightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

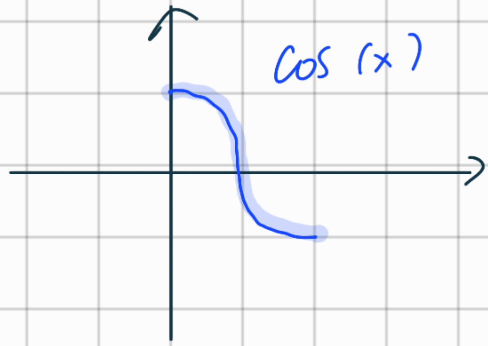
$\frac{1}{2}$



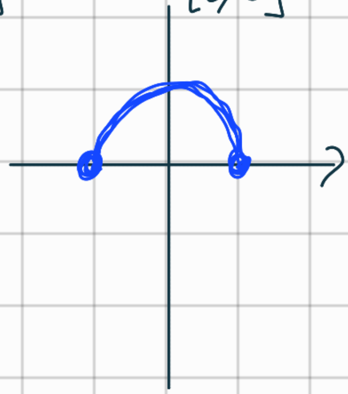


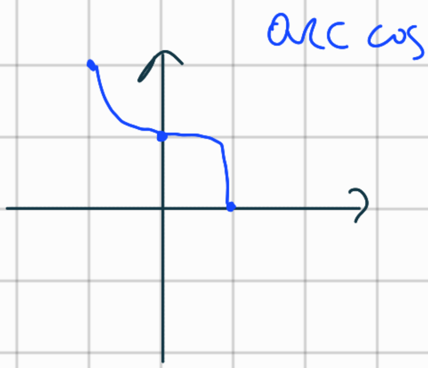
$\cos : [0, \pi] \longrightarrow [-1, 1]$

è INIETTIVA e SURIETTIVA



$\arccos \ [-1, 1] \longrightarrow [0, \pi]$

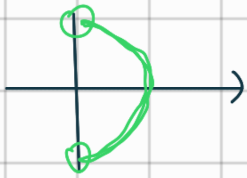




Regole 5 delle C.E.

$$\left.\begin{array}{l}\arcsin(f(x)) \\ \arccos(f(x))\end{array}\right. \Rightarrow f(x) \in [-1, 1]$$

$tg : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \longrightarrow \mathbb{R}$ è INIETTIVA e SURIETTIVA



$arctg : \mathbb{R} \longrightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$