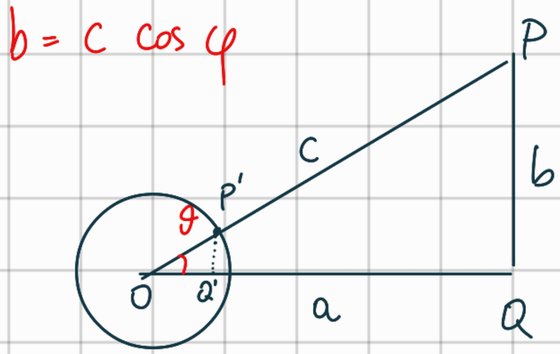
Lezione 12

20/11/23

$\overline{OP} = c$

$\overline{OQ} = a$

$\overline{PQ} = b$



$\overline{OP'} = 1$ perché è il raggio della circonferenza

$OP = c \cdot OP'$ $\qquad \overline{OQ'} = \cos\vartheta$

$\overset{\triangle}{OPQ}$ e $\overset{\triangle}{OP'Q'}$ sono triangoli simili

$\Downarrow$

$$OP : OP' = OQ : OQ'$$
$$c : 1 = a : \cos\vartheta$$

$$\begin{cases} a = c\cos\vartheta \\ b = c\sin\vartheta \end{cases}$$

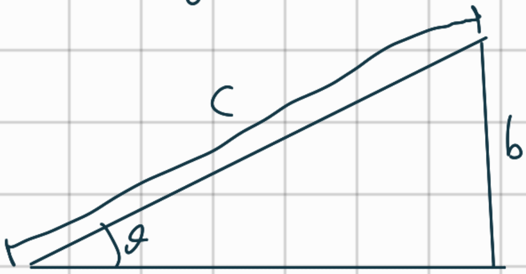
Un cateto è uguale al prodotto dell'ipotenusa per...

- il seno dell'angolo opposto
- il coseno dell'angolo adiacente

ESERCIZIO

C'è una rampa di cui conosciamo l'inclinazione $\vartheta = \frac{\pi}{6}$

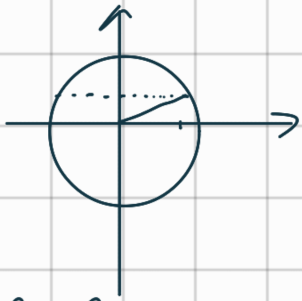
la lunghezza della rampa è 6 metri



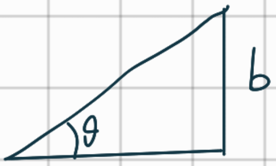
Domanda: di quanto si sale?

$\Downarrow$

$$b = c \sin\vartheta = 6m\left(+\frac{1}{2}\right) = 3m$$



Se il problema fornisce i due cateti e bisogna calcolare ϑ?



$$\begin{cases} b = c\sin\vartheta \\ a = c\cos\vartheta \end{cases}$$

$$\frac{b}{a} = \frac{c}{c}\frac{\sin\vartheta}{\cos\vartheta} = tg\vartheta$$

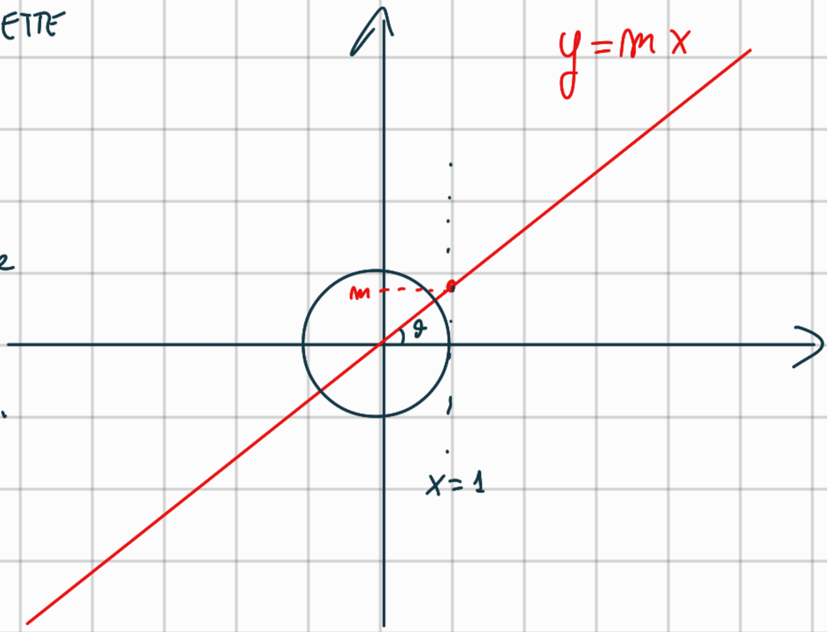
$\Downarrow$

$$\boxed{\vartheta = arctg\left(\frac{b}{a}\right)}$$

PROPRIETÀ DELLA TANGENTE E RETTE

$$m = tg\vartheta$$

Il coefficiente angolare è il valore della tangente dell'angolo che determina l'inclinazione della retta.

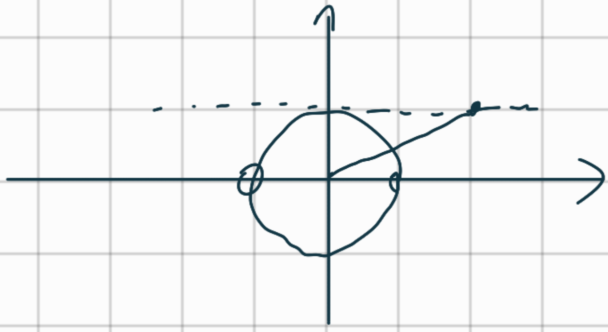


FORMULE DI ADDIZIONE DEL SENO e DEL COSENO

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$$

ALTRE FUNZIONI (MENO USATE)

$$ctg(x) = \frac{1}{tg(x)}$$

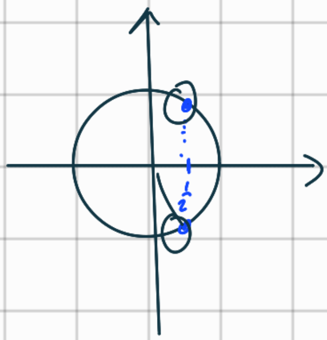


$$\sec(x) = \frac{1}{\cos x} \leftarrow \cos x \neq 0$$

$$\mathrm{cosec}(x) = \frac{1}{\sin x} \leftarrow \sin x \neq 0$$

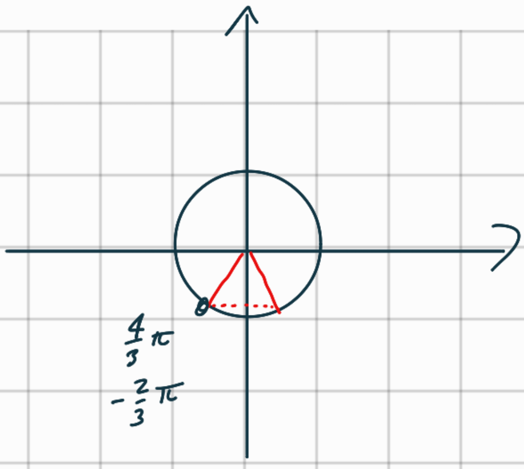
$$\cos(x) = \frac{1}{2}$$

$$\frac{\pi}{3} + 2k\pi \quad \cup \quad -\frac{\pi}{3} + 2k\pi \qquad k \in \mathbb{Z}$$



$$S = \left\{\dots, \underset{k=-1}{-\frac{5}{3}\pi}, \underset{k=0}{\frac{\pi}{3}}, \underset{k=1}{\frac{7}{3}\pi}, \underset{k=2}{\frac{13}{3}\pi}, \dots\right\} \cup \left\{\dots, \underset{k=-1}{-\frac{7}{3}\pi}, \underset{k=0}{-\frac{\pi}{3}}, \underset{k=1}{\frac{5}{3}\pi}, \underset{k=2}{\frac{11}{3}\pi}, \dots\right\}$$

$$\sin(x) = -\frac{\sqrt{3}}{2}$$

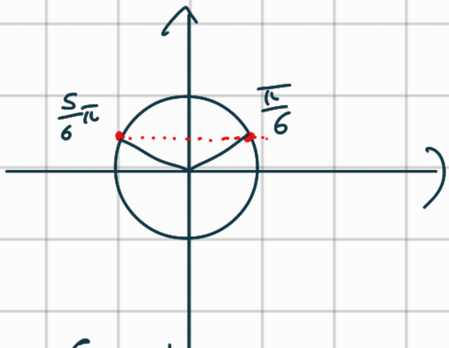


$$x = \frac{4}{3}\pi + 2k\pi \quad \vee \quad x = \frac{5}{3}\pi + 2k\pi$$

oppure

$$-\frac{2}{3}\pi + 2k\pi \qquad -\frac{\pi}{3} + 2k\pi$$

$$f(x) = \frac{5}{2\sin x - 1}$$



C.E.

$2\sin x - 1 \neq 0$

$2\sin x \neq 1$

$\sin x \neq \frac{1}{2} \rightarrow x \neq \frac{\pi}{6} + 2k\pi \wedge x \neq \frac{5}{6}\pi + 2k\pi$

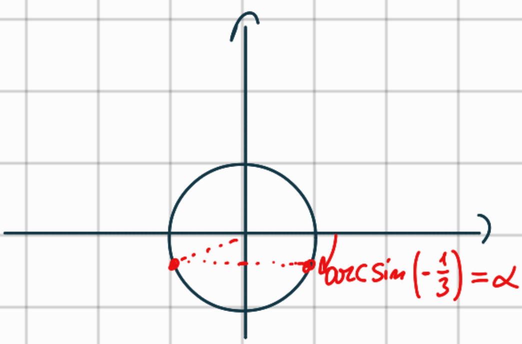
$$f(x) = \frac{3x+3}{\cos x - 2}$$

C.E.

$\cos x - 2 \neq 0$

$\cos x \neq 2 \rightarrow D = \mathbb{R}$

$$\sin x = -\frac{1}{3}$$



$x = \arcsin\left(-\frac{1}{3}\right) + 2k\pi$

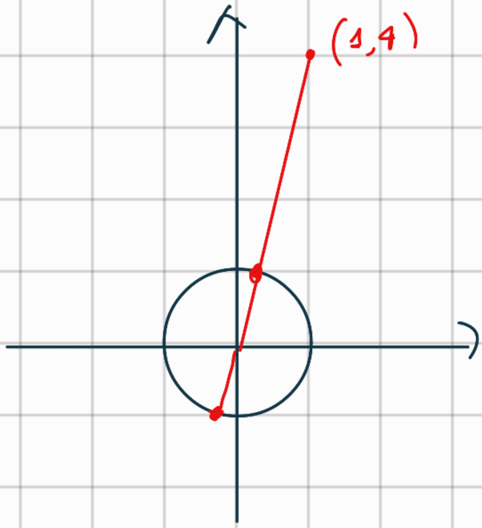
$\vee$

$x = \arcsin\left(-\frac{1}{3}\right) + \pi + 2k\pi$

$\arcsin\left(-\frac{1}{3}\right) + (2k+1)\pi$

- $tg(x) = 4$

$x = arctg(4) + k\pi \quad k \in \mathbb{Z}$



ANGOLI NOTEVOLI

α	$tg\,\alpha$
0	0
$\pi/6$	$\sqrt{3}/3$
$\pi/4$	1
$\pi/3$	$\sqrt{3}$
$\pi/2$	NON DEFINITA

$$\frac{\sin\left(\frac{\pi}{6}\right)}{\cos\left(\frac{\pi}{6}\right)} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

DOMINIO DI

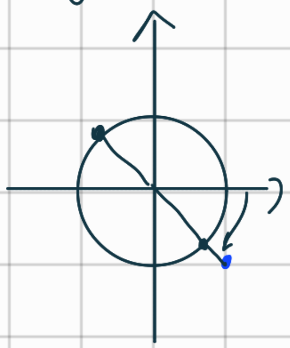
$$f(x) = \frac{\cos(x)}{tg^2(x) + 2tg(x) + 1}$$

$$\begin{cases} tg^2(x) + 2tg(x) + 1 \\ x \neq \frac{\pi}{2} + k\pi \end{cases}$$

1

4

$t = tg(x)$



$t^2 + 2t + 1 \neq 0$

$(t+1)^2 \neq 0$

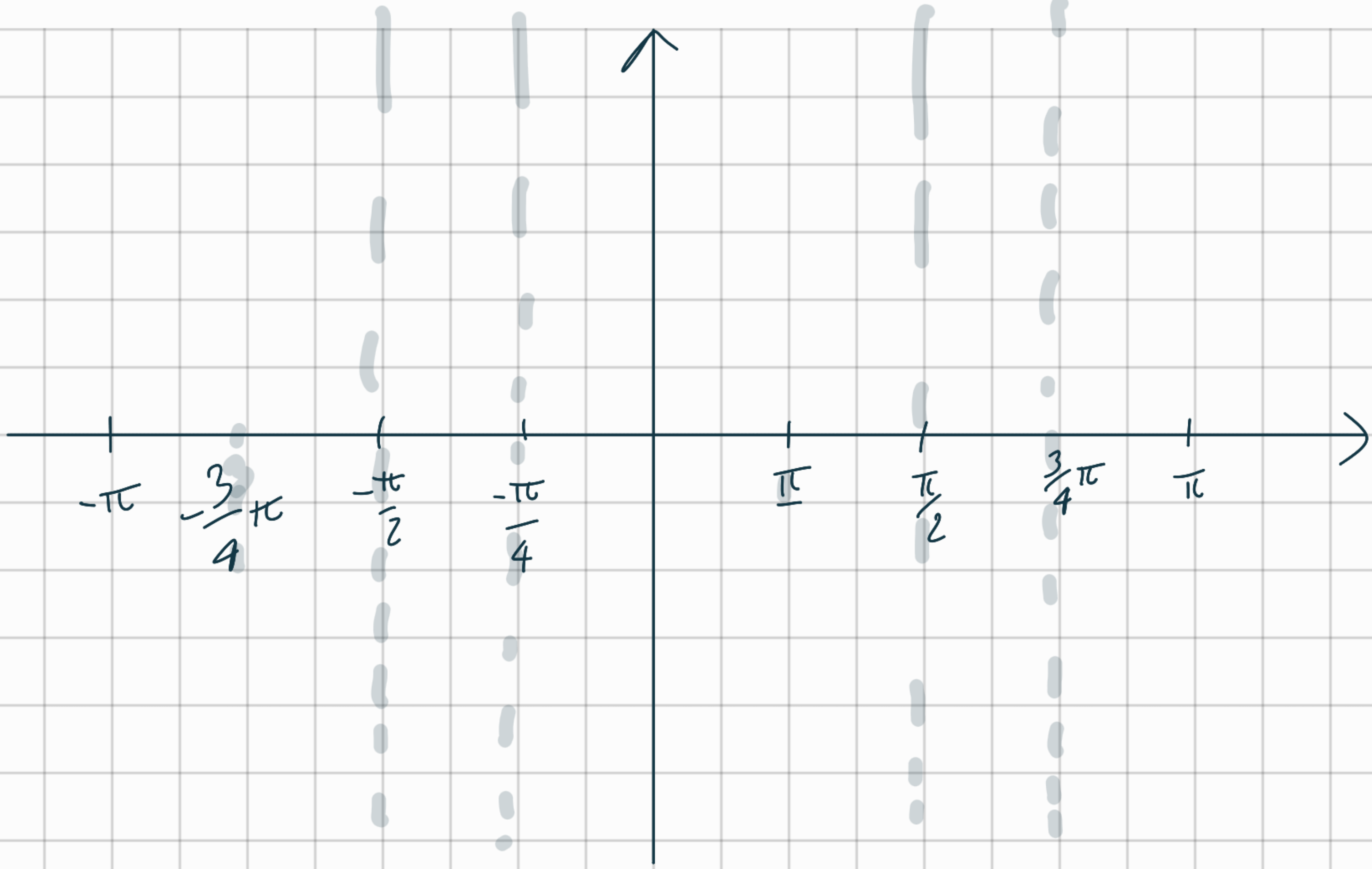
$t + 1 \neq 0$

$t \neq -1$

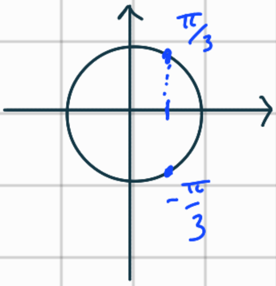
$tg(x) \neq -1 \quad \rightarrow \quad x \neq -\frac{\pi}{4} + k\pi$

$\downarrow k = 1$

$$-\frac{\pi}{4} + \pi = \frac{-1+4}{4} = \frac{3}{4}\pi$$



$$\cos(3x) = \frac{1}{2} \qquad x \in [-\pi, \pi]$$

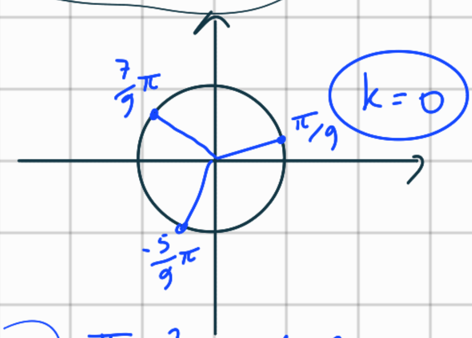


$$3x = \frac{\pi}{3} + 2k\pi \quad \vee \quad 3x = -\frac{\pi}{3} + 2k\pi$$

$$x = \frac{\pi}{9} + \frac{2}{3}k\pi$$

$k=1$ $\quad \frac{\pi}{9} + \frac{2}{3}\pi = \frac{1+6}{9}\pi = \frac{7}{9}\pi$

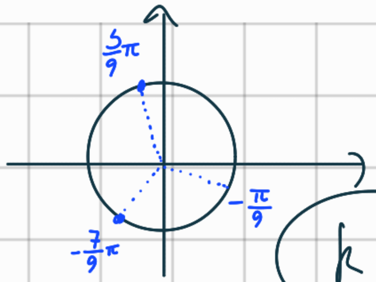
$$x = -\frac{\pi}{9} + \frac{2}{3}k\pi$$



$k=0$

$k=-1$ $\quad \frac{\pi}{9} - \frac{2}{3}\pi = \frac{1-6}{9}\pi = -\frac{5}{9}\pi$

PAGINA SEGUENTE



$k=0 \to x = -\frac{\pi}{9} \in [-\pi, \pi]$ ✓

$k=-1 \to x = -\frac{\pi}{9} - \frac{2}{3}\pi = \frac{-1-6}{9}\pi = -\frac{7}{9}\pi$

$k=1 = \ldots$ PER CASA $\ldots = \frac{5}{9}\pi$

$$S = \left\{ -\frac{7}{9}\pi, -\frac{5}{9}\pi, -\frac{\pi}{9}, \frac{\pi}{9}, \frac{5}{9}\pi, \frac{7}{9}\pi \right\}$$

Nell'intervallo richiesto ci sono 6 soluzioni