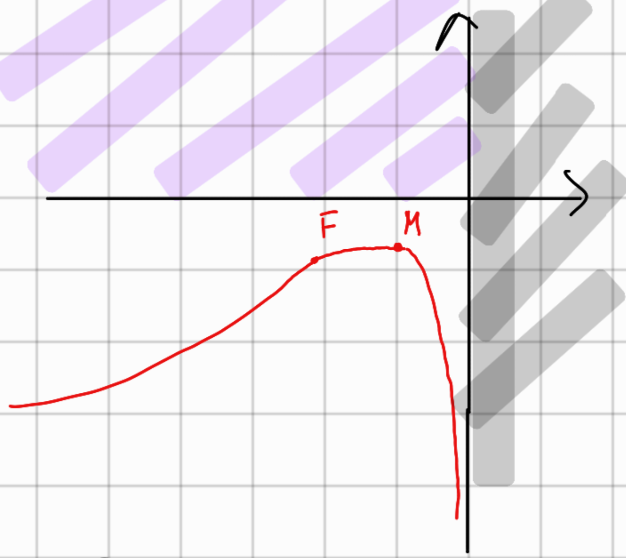
Lezione 31 e 32 17/01/24

ESAME BIOLOGIA SIMULAZIONE

$$f(x) = \ln\left(\frac{-x}{1+x^2}\right)$$

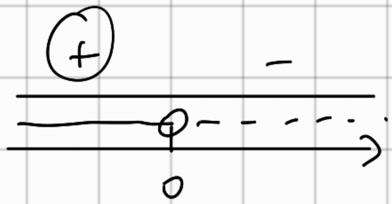


- DOMINIO

$$\begin{cases} 1+x^2 \neq 0 \implies \forall x \in \mathbb{R} \\ \frac{-x}{1+x^2} > 0 \end{cases}$$

$-x > 0 \quad \Downarrow \quad x < 0$

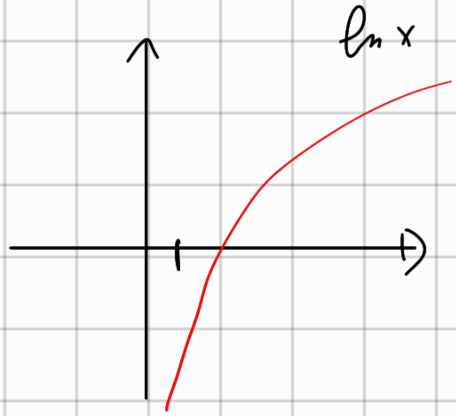
$1+x^2 > 0 \quad \Downarrow \quad \forall x \in \mathbb{R}$



$D = (-\infty, 0)$

- $0 \notin D \implies$ NON CALCOLO L'INTERSEZIONE ASSE Y

- STUDIO SEGNO E INTERSEZIONE ASSE X



$$\ln\left(\frac{-x}{1+x^2}\right) \geq 0 = \ln(1)$$

$$\Downarrow$$

$$\frac{-x}{1+x^2} \geq 1$$

$$\frac{-x}{1+x^2} - 1 \geq 0$$

$$\frac{-x-1-x^2}{1+x^2} \geq 0$$

$-x^2 - x - 1 \geq 0$

$x^2 + x + 1 \leq 0$

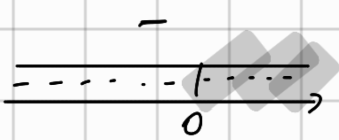
$\Delta = 1 - 4 = -3 < 0$

DISCORDI

$\emptyset$

$1+x^2 \geq 0$

$\mathbb{R}$



$f(x)$ è POSITIVA MAI
$f(x)$ è NULLA MAI
$f(x)$ è NEGATIVA $\forall x \in D$ $\quad \forall x \in (-\infty, 0)$

- LIMITI AGLI ESTREMI DEL DOMINIO

$$\lim_{x\to-\infty} \ln\left(\frac{-x}{1+x^2}\right) = \ln\left(\lim_{x\to-\infty} \frac{\overset{+\infty}{-x}}{\underset{+\infty}{1+x^2}}\right) = \ln(0^+) = -\infty$$

NON C'È L'A. OR.

$$\lim_{x\to-\infty} \frac{\overset{-\infty}{\ln\left(\frac{-x}{1+x^2}\right)}}{\underset{-\infty}{x}} \underset{DH}{=} \lim_{x\to-\infty} \frac{\frac{x^2-1}{-x(1+x^2)}}{1} = \lim_{x\to-\infty} \frac{\overset{+\infty}{x^2-1}}{\underset{+\infty}{-x^3-x}} = 0$$

NON C'È L'AS. OBL (poiché il limite dovrebbe essere $m \neq 0$)

$$\frac{d}{dx}\left[\ln\left(\frac{-x}{1+x^2}\right)\right] = g'(h(x))\,h'(x) = \frac{1}{\left(\frac{-x}{\cancel{1+x^2}}\right)} \cdot \frac{-1(1+x^2)+x(2x)}{(1+x^2)^{\cancel{2}}}$$

$g(h(x))$

$$= \frac{-1-x^2+2x^2}{-x(1+x^2)} = \frac{x^2-1}{-x(1+x^2)}$$

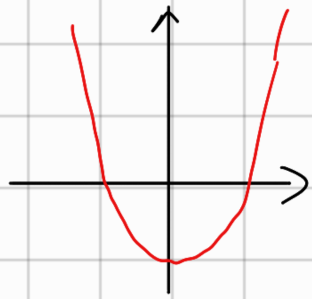
$$\lim_{x\to0^-} \ln\left(\frac{-x}{1+x^2}\right) \underset{\uparrow}{=} \ln \lim_{x\to0^-}\left(\frac{-x}{1+x^2}\right) = \ln(0^+) = -\infty$$

PERCHÉ $\ln(x)$ è CONTINUA

$x=0$ A.V. SINISTRO

STUDIO DEL SEGNO DELLA DERIVATA PRIMA E RICERCA DEI PUNTI STAZIONARI

$$f'(x) = \frac{x^2-1}{-x(1+x^2)} \geq 0$$



$x^2-1 \geq 0$	$-x > 0$	$1+x^2 > 0$
EQUAZIONE ASSOCIATA	$x < 0$	$\mathbb{R}$

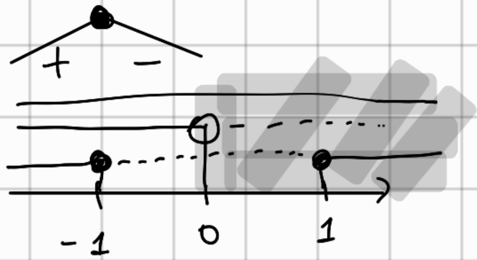
$x^2 - 1 = 0$

$x^2 = 1$

$x_{1,2} = \pm 1$

$(-\infty, -1) \cup (1, +\infty)$

SI ANNULLA IN $x_1 = -1$
$x_2 = 1$



f è CRESCENTE $x \in (-\infty, -1)$
f è DECRESCENTE $x \in (-1, 0)$
f è STAZIONARIA $x = -1$

$$M = \left(-1, \ln\left(\frac{1}{2}\right)\right) \simeq (-1, -0.69)$$

è punto di massimo

$$\ln\left(\frac{-(-1)}{1+(-1)^2}\right)$$

$$\ln\left(\frac{1}{2}\right) =$$

- STUDIO DELLA DERIVATA SECONDA E P.TI DI FLESSO

$$f'(x) = \frac{x^2-1}{-x(1+x^2)} = \frac{x^2-1}{-x^3-x}$$

$$f''(x) = \frac{2x(-x^3-x) - (x^2-1)(-3x^2-1)}{(-x^3-x)^2} = \frac{-2x^4-2x^2-[-3x^4-x^2+3x^2+1]}{(x^3+x)^2}$$

$$= \frac{-2x^4-2x^2+3x^4+x^2-3x^2-1}{(x^3+x)^2} = \frac{x^4-4x^2-1}{(x^3+x)^2}$$

$$\frac{x^4-4x^2-1}{(x^3+x)^2} \geq 0$$

$t = x^2$

t^2-4t-1

$\sqrt{20}=\sqrt{4}\cdot\sqrt{5}$
$=2\sqrt{5}$

$\Delta = 16+4=20$

$t_{12} = \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$

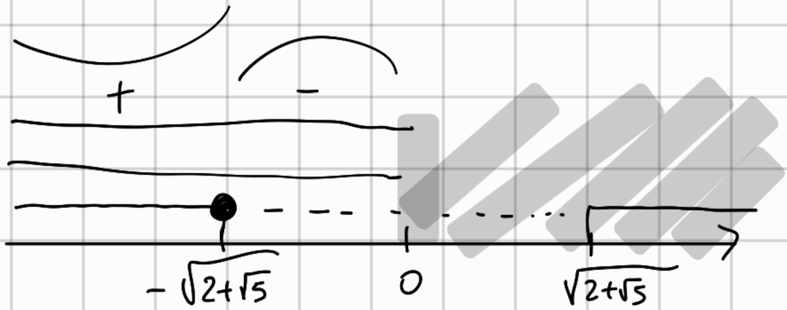
$(t-t_1)(t-t_2)$

$(t-2-\sqrt{5})(t-2+\sqrt{5})$

$\sqrt{5} > \sqrt{4} = 2$

$$\frac{(x^2-2-\sqrt{5})(x^2-2+\sqrt{5})}{(x^3+x)^2} \geq 0$$

$x^2-2-\sqrt{5} \geq 0$	$\underbrace{x^2-2+\sqrt{5}}$ ≥ 0	$(x^3+x)^2 > 0$
$x^2 \geq 2+\sqrt{5}$	$\forall x \in D$	$\forall x \in D$
$x_{12} = \pm\sqrt{2+\sqrt{5}}$		
$(-\infty, -\sqrt{2+\sqrt{5}}) \cup (\sqrt{2+\sqrt{5}}, +\infty)$		



$X_F = -\sqrt{2+\sqrt{5}}$

f è CONVESSA $x \in (-\infty, x_F)$

f è CONCAVA $x \in (x_F, 0)$

PRESENTA UN PUNTO DI FLESSO $F = (x_F, f(x_F)) \simeq (-2.06, -0.93)$

- $f'(-2) = \frac{(-2)^2-1}{2(1+(-2)^2)} = \frac{3}{2(5)} = \frac{3}{10}$

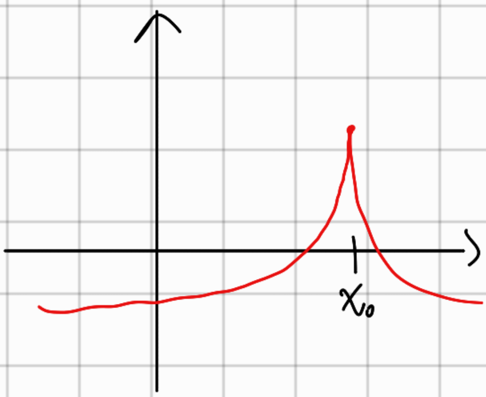
$$\frac{x^2-1}{-x(1+x^2)}$$

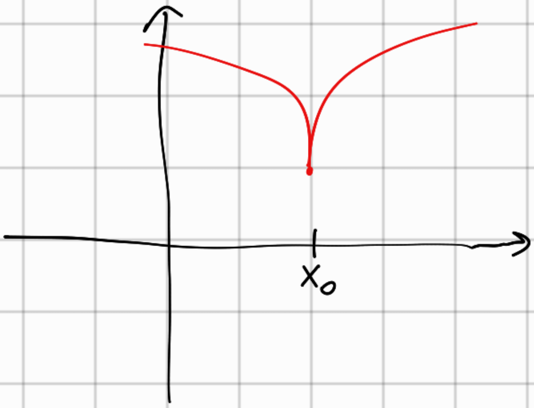
• DEFINIZIONE DI PUNTO DI CUSPIDE

Sie $f: D \to \mathbb{R}$ $\quad x_0 \in D$ SI DICE PUNTO DI CUSPIDE

SE $\lim_{x \to x_0^-} f'(x) = +\infty$ $\quad$ oppure $\quad$ $\lim_{x \to x_0^-} f'(x) = -\infty$

$\lim_{x \to x_0^+} f'(x) = -\infty$ $\qquad\qquad$ $\lim_{x \to x_0^+} f'(x) = +\infty$





RIPASSO

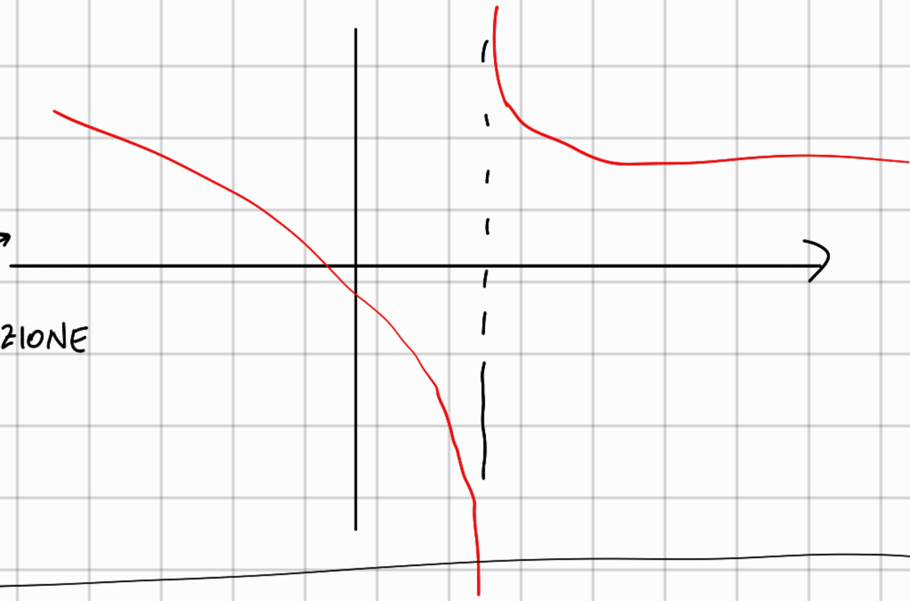
"$x_0 \in D$"

è FONDAMENTALE

ALTRIMENTI

GLI A.V. COME QUESTO

SODDISFEREBBERO LA DEFINIZIONE



ESERCIZIO 2 INTEGRALI

$$\int f'g = fg - \int fg'$$

$$f(x) = x^2 \ln x$$

- $$\int \underbrace{x^2}_{f'} \underbrace{\ln x}_{g}\, dx = \frac{x^3}{3}\ln(x) - \int \frac{x^{\cancel{3}\,2}}{3} \frac{1}{\cancel{x}}\, dx$$

$f' \to f = \frac{x^3}{3}$

$$= \frac{x^3}{3}\ln(x) - \frac{1}{3}\int x^2 dx = \frac{x^3}{3}\ln x - \frac{x^3}{9} + C$$

$$= \frac{x^3}{3}\left[\ln x - \frac{1}{3}\right] + C$$

- $$\int_1^e x \ln x\, dx = \left[\frac{x^3}{3}\left(\ln x - \frac{1}{3}\right)\right]_1^e = \frac{e^3}{3}\left(\ln e - \frac{1}{3}\right) - \frac{1}{3}\left(\ln(1) - \frac{1}{3}\right)$$

$1 - \frac{1}{3} = \frac{2}{3}$

$$= \frac{e^3}{3}\left(\frac{2}{3}\right) - \frac{1}{3}\left(0 - \frac{1}{3}\right)$$

$$= \frac{2e^3}{9} + \frac{1}{9} = \frac{1}{9}(2e^3 + 1)$$

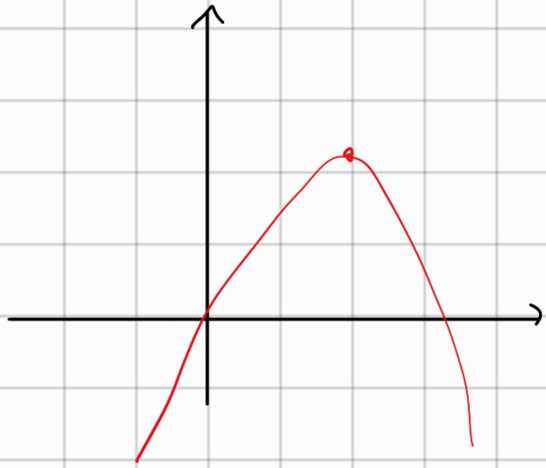
TEOREMA MEDIA INTEGRALE

$f : [a,b] \to \mathbb{R}$ CONTINUA $\Rightarrow \exists c$ t.c. $f(c)(b-a) = \int_a^b f(x)dx$

NO PERCHÈ $[1, e)$ NON È CHIUSO.

$$N(\mu) = 300\mu(6-\mu) = 1800\mu - 300\mu^2 = -300\mu^2 + 1800\mu$$

CALCOLARE IL PUNTO DI MASSIMO



- $N'(\mu) = 0$

$$N'(\mu) = -600\mu + 1800 = 0$$

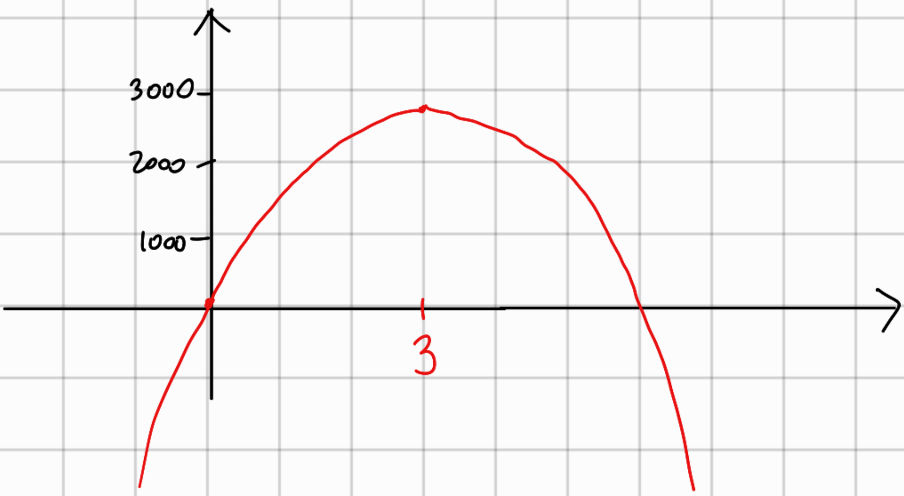
$$-600\mu = -1800$$

$$\mu = \frac{-1800}{-600} = 3$$

IL PUNTO DI MASSIMO SI HA PER $\mu = 3$

IN TALE PUNTO SOPRAVVIVERANNO $N(3)$ PICCOLI

$$N(3) = 300 \cdot 3(6-3) = 900(3) = 2700$$



DIAMETRO	54	29	14	42	37	1	X	(cm)
ETÀ	97	97	22	53	114	13	Y	(anni)

$$\bar{y} = \frac{1}{6}(97+97+22+53+114+13) = \frac{396}{6} = 66$$

MEDIANA 13 22 53 97 97 114

$$N \text{ PARI} = \text{MEDIANA}(Y) = \frac{y_{\frac{N}{2}} + y_{\frac{N}{2}+1}}{2} = \frac{53+97}{2} = \frac{150}{2} = 75$$

VARIANZA $\underbrace{\sigma_y^2 = \frac{1}{N}\sum_{i=1}^{N}(y_i-\bar{y})^2}$ =

97	97	22	53	114	13
31	31	-44	-13	48	-53

$$= \frac{1}{6}\left[(31)^2+(31)^2+(-44)^2+(-13)^2+(48)^2+(-53)^2\right]$$

$$= \frac{1}{6}\left[961+961+1936+169+2304+2809\right] = \frac{9140}{6} = \underline{1523.33} \text{ anni}^2$$

- CALCOLARE $\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$ $\qquad \sigma_x = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})^2}$

$$\sigma_{xy} = \frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})(y_i-\bar{y}) = \frac{1}{N}\sum_{i=1}^{N}x_i y_i - \bar{x}\bar{y}$$

$$\bar{x} = \frac{1}{6}\left[54+29+14+42+37+1\right] = \frac{177}{6} = 29.5$$

$(x_i-\bar{x})$	24.5	-0.5	-13.5	12.5	7.5	-28.5
	·	·	·	·	·	·
$(y_i-\bar{y})$	31	31	-44	-13	48	-53

$$\sigma_{xy} = \frac{1}{6}(759.5 \quad -15.5 \quad +682 \quad -162.5 + 360 + 1510.5) = \frac{3134}{6} = 522.33$$

$$\sigma_X^2 = \frac{1}{6}\left[(24.5)^2+(-0.5)^2+(-15.5)^2+(12.5)^2+(7.5)^2+(-28.5)^2\right]$$

$$= \frac{1}{6}\left[600.25+0.25+240.25+156.25+56.25+812.25\right]$$

$$= \frac{1}{6}\,1865.5 = 310.92$$

$$\rho_{xy} = \frac{522.33}{\sqrt{310.92}\cdot\sqrt{1523.33}} = \frac{522.33}{\underset{\sigma_x}{17.63}\cdot\underset{\sigma_y}{39.03}} = 0.759$$

TRA X Y C'È UNA RELAZIONE CONCORDE $\rho_{xy} > 0$

$|\rho_{xy}| > 0.7 \Rightarrow$ QUESTA RELAZIONE È FORTE

RETTA DI REGRESSIONE LINEARE

$$\alpha = \frac{\sigma_{xy}}{\sigma_X^2} \quad \frac{\cancel{cm}\cdot anni}{(cm)^{\cancel{2}}} \quad \frac{anni}{cm} \qquad \beta = \bar{y} - \alpha\bar{x}$$

X DIAMETRO

Y ETÀ

$$\alpha = \frac{522.33}{17.63} = 1.68$$

$$\beta = \bar{y} - \alpha\bar{x} = 66 - (1.68)(29.5) = 16.44$$

$$66 \text{ anni} - (\quad)\frac{anni}{\cancel{cm}}\cdot\cancel{cm} = 16.44 \text{ anni}$$

ETÀ

$$y = \alpha x + \beta = 1.68\cdot 10 + 16.44 = 16.8 + 16.44 = 33.24 \quad \text{ANNI}$$

[ALTERNATIVO]

H_0 = LE VISITE SONO INDIPENDENTI DALL'ESSERE SPORTIVO
H_1 =

REQUISITI : $F_{i,j} \geq 5$ ok

F_{ij}

SPORTIVI \ VISITE	SI	NO	
NO	15	45	60
QUALCHE VOLTA	10	15	25
SI	65	50	115
	90	110	200

$E_{i,j}$

SPORTIVI \ VISITE	SI	NO	
NO	$\frac{60 \cdot 90}{200} = 27$	33	60
QUALCHE VOLTA	11.25	13.75	25
SI	51.75	63.25	115
	90	110	200

$$\chi^{2\,*}_{(k-1)(h-1)} = \sum_{i=1}^{k} \sum_{j=1}^{h} \frac{(E_{ij} - F_{ij})^2}{E_{ij}}$$

$k = 2$
$h = 3$

$$\chi^{2\,*}_{2} = \frac{(27-15)^2}{27} + \frac{(33-45)^2}{33} + \frac{(11.25-10)}{11.25} + \frac{(13.75-15)^2}{13.75} + \frac{(51.75-65)^2}{51.75} + \frac{(63.25-50)^2}{63.25}$$

$$= 5.33 + 4.36 + 0.14 + 0.11 + 3.39 + 2.78 = 16.118 > 13.816$$

$\Downarrow$

LE DUE VARIABILI SONO DIPENDENTI L'UNA DALL'ALTRA AL 99.9 %