Esercizi sulle equazioni differenziali

Equazioni Separabili del tipo $u' = g(x)u$

1. $u' + u = 0$;
2. $u' - 4u = 0$;
3. $u' + x^2 u = 0$;
4. $u' - (u/x) = 0$ per $x > 0$;
5. $u' = (1 - x^2)^{-1/2} u$;
6. $u' = \dfrac{3x - 2}{x^2 - 2x - 8} u$ per $-2 < x < 4$;
7. $u' = \dfrac{2x + 1}{x^2 - 2x - 1}$ per $x < 1$;
8. $u' = (\ln x)u$ per $x > 0$;
9. $u' + 2u = 0$, $u(0) = 4$;
10. $u' = (u/2)$, $u(2) = -1$;
11. $u' = \dfrac{x}{x^2 + 4x - 5} u$, $u(0) = 1$;
12. $u' + x(\ln(2x))u = 0$, $u(1/2) = 0$;
13. $u' + \sqrt{1 + 2x}\, u = 0$, $u(4) = 10$;
14. $u' = \dfrac{1}{x(1 - x)} u$, $u(1/4) = 1$.

Equazioni Lineari del Primo Ordine

15. $u' + 2u = x$;

16. $u' - u = \text{sen}\,(2x)$;

17. $u' + 2xu = 4x$;

18. $x^2u' + xu + 1 = 0$;

19. $xu' - 4u = x + 3x^2$;

20. $u' + (\text{cotg}\,x)u = 3\text{sen}\,x\cos x$;

21. $xu' + \dfrac{1}{\ln x}u = 1$ per $x > 0$;

22. $u' = \dfrac{1}{\cos x}u$, $u(\pi/4) = (1/\sqrt{2})$;

23. $u' + |x|u = 0$, $u(0) = 4$;

24. $u' + u = 2x + 5$, $u(0) = 4$;

25. $u' + 4u = 6\,\text{sen}\,(2x)$, $u(0) = -3/5$;

26. $u' - 2u + e^{2x} = 0$, $u(0) = 2$;

27. $u' - 2u = x^2e^{2x}$, $u(0) = 2$;

28. $u' - 2xu - x = 0$, $u(0) = 1$;

29. $e^u u' + e^u = 4\,\text{sen}\,x$, $u(0) = 0$ (Consiglio: Sia $v = e^u$);

30. $(x^2 + 1)u' - 2xu = x^2 + 1$, $u(1) = \pi$;

31. $u' - (\text{tg}\,x)u = e^{\text{sen}\,(x)}$, $0 < x < \pi/2$, $u(1) = 0$;

32. $u' + 2xu = e^{-x^2}$, $u(0) = 1$;

33. $u' - 2u = x^2$, $u(0) = 2$;

34. $u' + (2u/x) = (\cos x)/x^2$, $u(\pi) = 0$, $x > 0$;

35. $u' + 3u = 6x$;

36. $u' - u = 4e^{2x}$;

37. $u' - (2u/x) = \operatorname{sen} x$;

38. $u' - \dfrac{x}{x^2+1}u = 1$;

39. $u' - (\operatorname{tg} x)u = 1$;

40. $u' + (u/x) = 2 + (1/x^2)$.

Equazioni separabili

41. $u' = (x^2/u)$;

42. $u' = x^2\sqrt{u}/(1+x^2)$;

43. $u' = e^{x+u}$;

44. $u' = \dfrac{x - xu^2}{u + x^2u}$;

45. $xu' = u^2 - 3u + 2$;

46. $u' = \dfrac{x+1}{u^4+1}$;

47. $u' = xe^x/(2u)$;

48. $u' = 1/(\operatorname{tg} x \cos^2 u)$;

49. $x^2uu' = u - 1$;

50. $u' = \dfrac{\ln x \cos u}{x \operatorname{sen} 2u}$;

51. $u' = (\operatorname{sen} x)/u$, $u(0) = -1$;

52. $u' = x^2e^{-u}$, $u(0) = 2$;

53. $u' = 1 - u^2$, $u(0) = 0$;

54. $u' = (u^2+u)/x$, $u(1) = -1$;

55. $u' = \dfrac{3x^2+2}{2(u-1)}$, $u(0) = -1$;

56. $u' = \sqrt{1-u^2}$, $u(\pi/2) = 0$;

57. $x^2u' - \dfrac{1}{2}\cos 2u = \dfrac{1}{2}$, $u(1) = \pi/4$;

58. $u' = xu^4\sqrt{1+3x^2}$, $u(1) = -1$;

59. $u' = -3u^{4/3}\text{sen}\, x$, $u(\pi/2) = (1/8)$;

60. $u' + \dfrac{4u}{1-x^2} = 0$, $u(2) = 9$.

Equazioni Lineari Omogenee del Secondo Ordine

61. $u'' + 5u' + 6u = 0$;

62. $u'' - 10u' + 25u = 0$;

63. $u'' + 2u' + 5u = 0$;

64. $u'' - 2u' + 5u = 0$;

65. $u'' + 3u' - 10u = 0$;

66. $u'' + 9u = 0$.

Equazioni Lineari Non Omogenee del Secondo Ordine

67. $u'' - u' - 2u = x$;

68. $u'' - 4u = 6e^x$;

69. $u'' - 2u' + u = 4\,\text{sen}\, x$;

70.* $u'' + 5u' + 4u = 2e^{-x}$;

71. $u'' = x^2 + x + 1$;

72. $u'' + 2u' + 5u = 10x^2 - e^{-2x}$;

73.* $u'' + 4u = x - 2\,\text{sen}\, 2x$;

74.* $u'' + u = x\,\cos 2x$;

75. $u'' + u' - 4u = 2\,\text{senh}\,x$;

76. $u'' + u = 8\,\cos^3 x$;

77. $u'' - u' - 2u = x$, $u(0) = 0$, $u'(0) = 1$;

78. $u'' - 4u = 6e^x$, $u(0) = 1$, $u'(0) = 2$;

79. $u'' - 2u' + u = 4\,\text{sen}\,x$, $u(0) = 1$, $u'(0) = 0$;

80.* $u'' + 5u' + 4u = 2e^{-x}$, $u(0) = 1$, $u'(0) = 1$;

81.* $u'' + 4u = x - 2\,\text{sen}\,2x$, $u(\pi) = 0$, $u'(\pi) = 1$;

82. $u'' + u = 8\,\cos^3 x$, $u(\pi/6) = 0$, $u'(\pi/6) = 0$.

83. $u'' - u = 2e^{3x}$;

84. $u'' + 9u = 3x + 2$;

85. $u'' - 7u' + 12u = 5e^{4x}$;

86.* $u'' - 4u = 3e^{-2x} - x^2$;

87.* $u'' + 4u = 4\,\cos^3 2x$.

88. $u'' - u = 2e^{3x}$, $u(0) = 1$, $u'(0) = 2$;

89. $u'' + 9u = 3x + 2$, $u(0) = 0$, $u'(0) = 1$.

90.* $u'' + \omega^2 u = f(x)$, $u(0) = u'(0) = 0$.

91. $u'' + 4u' + 13u = 6\,\cos 2x - 5\,\text{sen}\,2x$;

92. $u'' + 3u' + 2u = \cos 3x + 2\,\text{sen}\,3x$;

93. $u'' + 4u' + 5u = 7\,\cos x + 8\,\text{sen}\,x$;

94. $4u'' + 12u' + 9u = 3\,\cos x$;

95. $3u'' + 13u' + 4u = 4\,\cos 2x - \text{sen}\,2x$;

96. $u'' + cu' + u = A\,\cos x$, $c = 2, 0.2, 0.002$;

97. $3u'' + 12u = 5\,\cos 3x$;

98. $4u'' + 36u = 3\,\text{sen}\,2x$;

99. $u'' + \pi^2 u = 2\,\text{sen}\,(22x/7)$;

100.* $u'' + u = A\,\cos(1+\epsilon)x$, $\epsilon = 1, 0.1, 0.001$.

Equazioni Lineari Omogenee di Ordine $n \geq 3$

101. $u''' + 3u'' + 2u' = 0$;

102. $u''' + 10u'' + 29u' = 0$;

103. $u''' - 6u'' + 9u' = 0$;

104. $u''' + 8u = 0$;

105. $u''' + 13u'' = 0$;

106. $u''' - 27u = 0$;

107. $u^{(4)} + 5u''' + 6u'' = 0$;

108. $u^{(4)} - 8u''' + 17u'' = 0$;

109. $u^{(4)} + 3u''' + 3u'' + u' = 0$;

110. $u^{(5)} - 3u^{(4)} + 3u''' - u'' = 0$;

111. $u^{(4)} - 5u'' + 4u = 0$;

112. $u^{(4)} + 5u'' + 4u = 0$;

113. $u^{(4)} + 6u'' + 9u = 0$;

114. $u^{(5)} + 8u''' + 16u' = 0$;

115. $u^{(4)} - 81u = 0$;

116. $u^{(5)} + u' = 0$;

117. $u^{(4)} - 6u'' + 9u = 0$;

118. $u^{(4)} + 6u''' + 13u'' = 0$;

119. $u^{(6)} - 64u = 0$;

120. $u^{(6)} + u = 0$.