

Trasformate di Fourier di alcune funzioni elementari

	$f(x)$	$F(k)$
impulso esponenziale	$e^{-ax}H(x) \quad (a > 0)$	$\frac{1}{a + \mathbf{i}k}$
	$e^{ax}H(-x) \quad (a > 0)$	$\frac{1}{a - \mathbf{i}k}$
impulso pari	$e^{-a x } \quad (a > 0)$	$\frac{2a}{a^2 + k^2}$
impulso dispari	$e^{-ax}H(x) - e^{ax}H(-x) \quad (a > 0)$	$-\frac{2\mathbf{i}k}{a^2 + k^2}$
onda quadra	$c [H(x + a) - H(x - a)]$	$2ac \frac{\sin ak}{ak}$
delta di Dirac	$\delta(x)$	1
Gaussiana	e^{-x^2}	$\sqrt{\pi}e^{-\frac{k^2}{4}}$

Proprietà della trasformata di Fourier

1. $\mathcal{F}\{\alpha f + \beta g\} = \alpha \mathcal{F}\{f\} + \beta \mathcal{F}\{g\}$
2. $\mathcal{F}\{f(x - x_0)\} = e^{-\mathbf{i}x_0 k} F(k)$
3. $\mathcal{F}\{e^{\mathbf{i}k_0 x} f(x)\} = F(k - k_0)$
4. $\mathcal{F}\{f(ax)\} = \frac{1}{|a|} F\left(\frac{k}{a}\right)$
5. $\mathcal{F}\{g\} = f \implies \mathcal{F}\{f\} = 2\pi g(-k)$
6. $\mathcal{F}\{f(x) \cos k_0 x\} = \frac{1}{2} [F(k - k_0) + F(k + k_0)]$
7. $\mathcal{F}\{f(x) \sin k_0 x\} = \frac{1}{2\mathbf{i}} [F(k - k_0) - F(k + k_0)]$
8. $\mathcal{F}\{f'\} = \mathbf{i}kF(k), \quad \mathcal{F}\{f^{(n)}\} = (\mathbf{i}k)^n F(k)$
9. $\mathcal{F}\{xf(x)\} = \mathbf{i}F'(k), \quad \mathcal{F}\{x^n f(x)\} = \mathbf{i}^n F^{(n)}(k)$
10. $\mathcal{F}\{f * g\} = F(k)G(k), \quad \mathcal{F}^{-1}\{F(k)G(k)\} = f * g$