Project on

Numerical Linear Algebra

(PhD Course)

Title: **Image Compression using SVD**

By:

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1 Introduction

In recent technology life, images are used in many computer applications. In this manner, storing and transmitting them are difficult with respect to limitation of storage devise. One of the main use of the images are in image processing task, specially for surveillance systems. In this kind of systems huge amount of images are acquiring due to the embedded camera in variant places (e.g. airport hall). The acquired images must be analysed in the same time too, but for a high quality images this task becomes as a time consuming task. One of the possible solution for this kind of application is using image compression to achieve a significant trade-off between the quality of image and its storage space. Some compression techniques always remove the redundant data from a given image. It is worth to point out that compressing an image also effect on its quality.

In this report, Singular Value Decomposition (SVD) is utilized to compress and reduce the storage space of an image.

2 Problem formulation

Let I a given image to be compressed. While I is a gray scale image and let say it is a square image, denoted as $I \in F^{N\times N}$, where F represents the real numbers. The SVD I is defined as

$$I = USV^H \tag{1}$$

where $U \in F^{N\times N}$ and $V \in F^{N\times N}$ are unitary matrices, and $S \in F^{N\times N}$ as a diagonal matrix, $S = diag(\alpha_1, \dots, \alpha_r, 0, \dots, 0)$. The singular values are ordered in decreasing order, $\alpha_1 \geq \dots \geq \alpha_r \geq 0$. Accordingly, in many applications, it can be useful to approximate I with low-rank matrix

$$I = \begin{bmatrix} U_r & U_{n-r} \end{bmatrix} \begin{bmatrix} S_r & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_r^H \\ V_{n-r}^H \end{bmatrix} \tag{2}$$

Then, we have

$$I = U_r S_r V_r^H = I_r \tag{3}$$

Also, the corresponding error for the compressed image can be computed as the below equation with respect to Frobenius norm of the error

$$E = \frac{\|I - I_r\|_F^2}{\|I\|} \tag{4}$$

Figure 1: Lena original image

Table 1: Storage space of the compressed images (in variant number of singular values of r) and the original image (all reported in KilloByte).

Originl	$r = 10$	$r = 30$	$r = 50$	$r = 70$	$r = 90$	$r = 110$
154	26	33	37	38	39	39

3 Implementation details and experimental results

To demonstrate the effect of SVD on compression I was evaluated on a sample image of *Lena* (See figure 1), which normilized in 512×512 pixels ($n \times n$). To the given image, the best $r - approximation$s are evaluated in the ranges of $[10 \dots 110]$ with 20 step size as the number of the first singular values., and the goal consists of achieving a significant trade-off between the image quality and storage space of the given image.

Figure 2 show the behaviour of the given image in variant value of r. Also, figure 3 the error comparison of the compressed images with the original image, and the change in error with increasing number of singular values showed in the figure.

Table 1 shows the storage space of the compressed images and the original image. The best r-approximation (figure 2(f)) with $r = 110$ shows the quality of the image is satisfactory, and informative features can be extracted as same as the original image. In this respect, the storage space for that rank has been reduced to 75%.

(a) $r = 10$ (b) $r = 30$ (c) $r = 50$

(d) $r = 70$ (e) $r = 90$ (f) $r = 110$

Figure 2: Results on variant value of r-approximation.

4 Conclusion

In this project, the process of the image compression using SVD has been presented. It is observed that the matrix coefficients as the ranked of the image matrix increases which indicates that the maximum energy is concentrated with only first few coefficients. It is also worth to point out that with increasing in the rank , the number of entries in the matrix would be increased which in fact improve the perceptual quality of the image, accordingly.

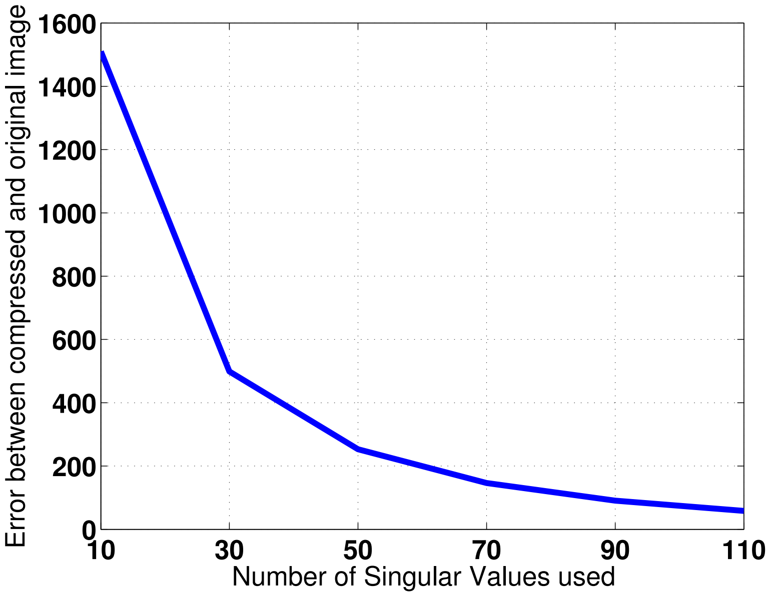

Figure 3: Error comparison.