Università degli Studi di Cagliari
Electrical and Electronic Department

Multi-Objective Optimization of the Gate Circuit

Student: Milad Moradpour

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1. Introduction

In electric vehicles, a DC-DC power converter supply energy from a DC source to the drive system. The bridge connection of two transistors is a very common topology in DC-DC converter circuits (fig. 1). Each transistor has a gate circuit including a resistor. The designer should evaluate the amount of the resistor. The resistance effects both efficiency and electromagnetic interference (EMI) of the converter. The aim of this project is to evaluate the resistance of each gate circuit (two decision variables) in such a way to maximize the efficiency (or equally minimize the negative amount of efficiency) and minimize the EMI (two objective functions).

Two perform an optimization, the mathematical formulation of the objective functions and constraint functions are needed. Constraint functions are inequality lower/upper bounds in which the decision variables can be continuously changed. But, there is no mathematical model of decision variables as the inputs and objective functions as the outputs. To find the models, a reasonable amount of experiments has been done and inputs/outputs are obtained as series data. Then, a mathematical model has been fitted to the data with a multi-variable regression method since there are two variables. In statistics and machine learning toolbox of MATLAB there are some solvers such as fitlm based on least squares method to perform the regression. The obtained fitted functions predict the output very well.

Since the fitted models (objective functions) are non-linear and quadratic, a sequential quadratic programming (SQP) method should been used to minimize the objective functions. The obtained results using fmincon, which is a solver in optimization toolbox of MATLAB, are satisfying. In addition, since there are two objective functions, Pareto Front of objectives should be obtained. Thereafter, the designer is able to do his own trade off based on his needs using Pareto front. In global optimization toolbox of MATLAB, there is gamultiobj solver, which is based on genetic algorithm optimization applicable for nonlinear multi-objective fitness functions.
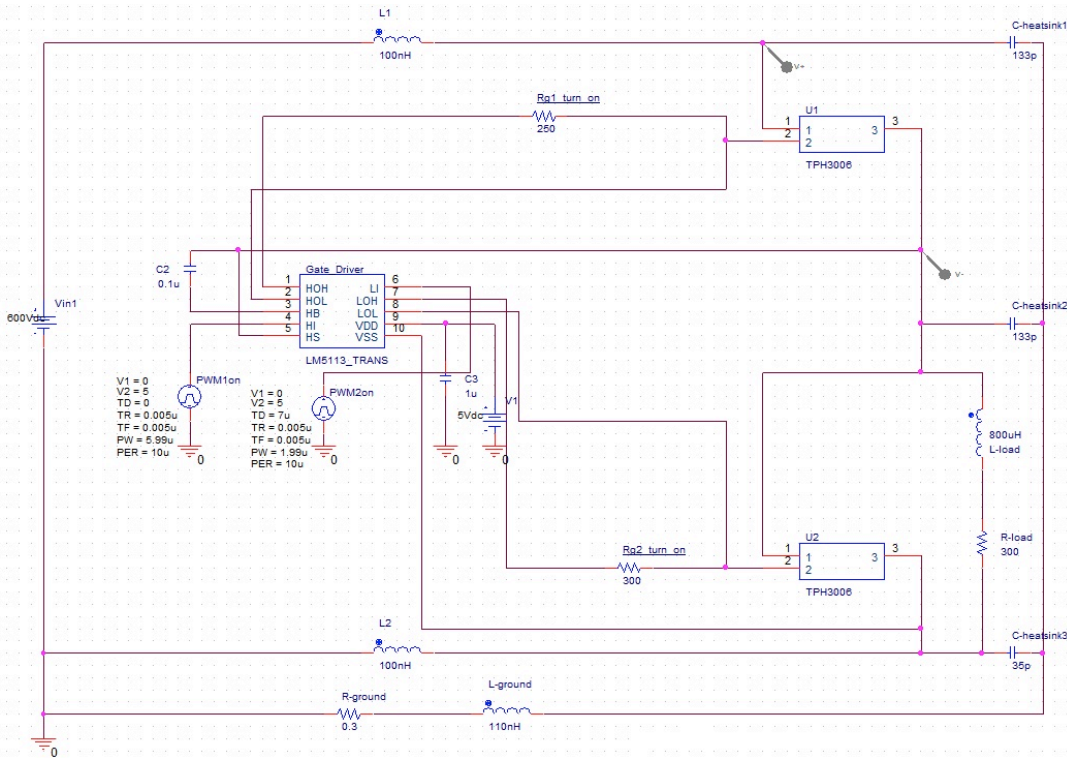
Figure 1. Schematic of the bridge circuit. Decision variables are shown by Rg1_turn_on and Rg2_turn_on. Efficiency is defined as the percentage of the input power generated by Vin1, which is used by the load (L-load and R-load). EMI is the amount of ground current (L-ground or R-ground) in dBuA dimension

2. Multi-Variable Regression

As it can be seen in fig. 1, rather than Rg1_turn_on and Rg2_turn_on, it is possible to add also Rg1_turn_off and Rg2_turn_off for the switching off state of

transistors U1 and U2. It is possible also to add an inductor in series with on resistors to obtain better performance for the circuit. In general, more decision variables provide higher degree of freedom for the designer. But, it also causes complexity and possibly instability for the system. Considering four decision variables, Rg1_turn_on, Rg2_turn_on, R_turn_off, and L1_turn_on, a multivariable regression has been accomplished by fitlm. The results are two quadratic functions for efficiency and EMI as it is shown in fig. 2:

a) EMI (left Command Window):

```
Linear regression model:
    y ~ [Linear formula with 15 terms in 4 predictors]

Estimated Coefficients:
                 Estimate         SE          tStat       pValue
                _____   _____   _____   _____

    (Intercept)          0             0         NaN          NaN
    x1                   0             0         NaN          NaN
    x2                   0             0         NaN          NaN
    x3              9.8871      0.021798      453.58    3.7653e-36
    x4                   0             0         NaN          NaN
    x1:x2       8.1328e-07    2.0431e-05    0.039807       0.96871
    x1:x3                0             0         NaN          NaN
    x1:x4                0             0         NaN          NaN
    x2:x3       -0.0013191    0.00021763     -6.0613    1.2712e-05
    x2:x4         0.019875    7.9434e-05      250.21    9.2689e-32
    x3:x4         -0.19511    0.00037359     -522.25    3.4277e-37
    x1^2       -2.5262e-06    2.1665e-06      -1.166        0.2597
    x2^2         9.833e-06    2.2069e-06      4.4556    0.00034723
    x3^2       1.9919e-07    2.2091e-06    0.090164       0.92921
    x4^2        -0.0003937    0.00011689     -3.3682     0.0036502

Number of observations: 32, Error degrees of freedom: 23
Root Mean Squared Error: 0.504
R-squared: 0.729,  Adjusted R-Squared 0.635
F-statistic vs. constant model: 7.75, p-value = 5.13e-05
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b) efficiency (right Command Window):

```
Linear regression model:
    y ~ [Linear formula with 15 terms in 4 predictors]

Estimated Coefficients:
                 Estimate         SE          tStat       pValue
                _____   _____   _____   _____

    (Intercept)          0             0         NaN          NaN
    x1                   0             0         NaN          NaN
    x2                   0             0         NaN          NaN
    x3              3.7595      0.074308      50.594    5.5764e-20
    x4                   0             0         NaN          NaN
    x1:x2      -2.0287e-05    6.9646e-05    -0.29129       0.77435
    x1:x3                0             0         NaN          NaN
    x1:x4                0             0         NaN          NaN
    x2:x3       -0.0059658    0.00074188     -8.0415    3.4015e-07
    x2:x4        0.0051062    0.00027079      18.857    7.8114e-13
    x3:x4        -0.063422     0.0012736     -49.799    7.2863e-20
    x1^2       -3.5317e-06    7.3853e-06    -0.47821        0.6386
    x2^2        4.0644e-05    7.5232e-06      5.4025    4.7635e-05
    x3^2        5.8266e-06    7.5308e-06      0.7737       0.44973
    x4^2        0.00027599    0.00039846     0.69264       0.49789

Number of observations: 32, Error degrees of freedom: 23
Root Mean Squared Error: 1.72
R-squared: 0.92,  Adjusted R-Squared 0.893
F-statistic vs. constant model: 33.2, p-value = 7.52e-11
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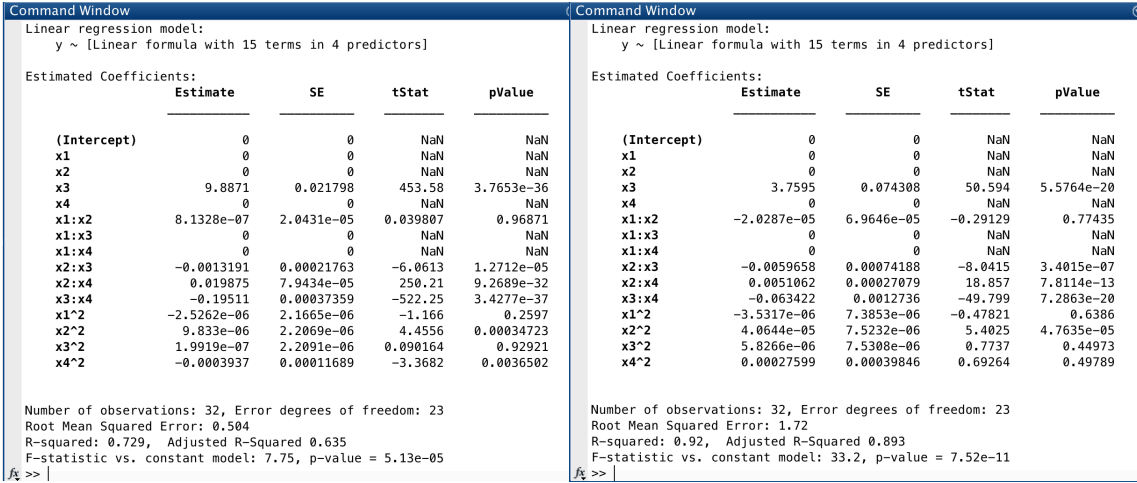
Figure 2: a) Quadratic fitted function of EMI with four variable   b) Quadratic fitted function of efficiency with four variables

Since there are four variables it is not possible to plot the experiments and compare them with fitted functions. But, the prediction response of fitted function to arbitrary inputs is only satisfying when the inputs are close to the experiments. Otherwise, the response is far from the reality. For example, the efficiency fitted function output response to Rg1_turn_on = 100 Ohm, Rg2_turn_on = 1000 Ohm, R_turn_off = 10 Ohm, and L1_turn_on = 100 micro farad (the upper/lower bounds are 1k/0.1 Ohm for the resistors and 0.1/100 micro farad for the inductor) is -9560.6 which has no sense since the efficiency is always between zero to one hundred.

The fitted functions can't precisely model the circuit because there are not enough experiments to perform the regression. In this example, 32 experiments have been done and then inputs-outputs series of data has been used for the regression. However, a reasonable number of experiments could be $10 \times 10 \times 10 \times 10 = 10000$ (10 points for each decision variables in feasible region).

Because it is not possible to perform 10 thousands experiments and collect data to have a successful regression, an alternative is to simplify the problem by fixing some of the decision variables and reduce the number of them. To do this, the variable with low sensitivity to the outputs has been investigated by circuit analysis (as it can be seen in fig. 1, R_turn_off is equal to zero). Moreover, the inductor is supposed to be added to Rg1_turn_on only after evaluating and fixing of Rg1_turn_on and Rg2_turn_on. In this way, only two decision variables has been left. Repeating the procedure with 25 experiments, 5 points for each variable with [10 50 100 200 300] Ohm, lead to a satisfying results as can be seen in fig. 3. Since there are two inputs and one output respectively, a 3D diagram can be plotted to compare experiment results and the fitted functions.
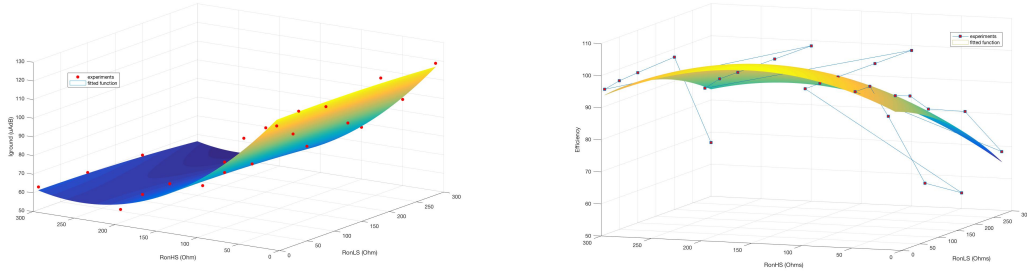
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Figure 3: a) Quadratic fitted function of EMI with two variables | b) Quadratic fitted function of efficiency with two variables

3. Multi-Objective Non-Linear Quadratic Optimization

Before performing the multi-optimization using gamultiobj, an optimization has been performed separately for each objective function to evaluate the accuracy of the regression modeling. To do this, fmincon has been selected from the same toolbox, optimization, in MATLAB. fmincon, which is suitable for non-linear quadratic single variable functions, uses SQP method to solve the optimization problem.

Objective functions, which has been found in section 2 by multivariable regression method are EMI and efficiency respectively:

$$F_1(x) = EMI(R_1, R_2) \tag{1}$$
$$= 123.3 + 0.0088362 \times R_1 - 0.53186 \times R_2$$
$$- 2.6687 \times 10^{-5} \times R_1 \times R_2 - 4.7908 \times 10^{-5} \times R_1{}^2$$
$$+ 0.0010733 \times R_2{}^2$$

$$F_2(x) = EFF(R_1, R_2) \tag{2}$$
$$= 91.622 - 0.03835 \times R_1 + 0.15262 \times R_2$$
$$+ 0.00020793 \times R_1 \times R_2 - 0.00018997 \times R_1{}^2$$
$$- 0.00049047 \times R_2{}^2$$

Using fmincon with any initial point (such as [75 75]) in defined feasible region (feasible region is defined as lower/bound inequalities, 10 to 300 Ohm) returns [300 259.5] as the inputs to reach minimum of $EMI(R_1, R_2)$ and [10 157.7] as the inputs to reach minimum of $-EFF(R_1, R_2)$ (since we like to maximize efficiency, the problem can be defined as minimization of the negative amount of efficiency).

Using fitted function $F_1(x)$, where x is the vector of input variables as $[R_1, R_2]$, to predict the outputs returns $EMI(300,259.5) = 53.7515\ [\mu AdB]$ while performing an experiment with the same inputs results in $61.4545\ [\mu AdB]$. On the other hand, the range of EMI for the 25 initial experiments, which has been used to accomplish the regression, is [53.3293 120.8728].

Using fitted function $F_2(x)$, where x is the vector of input variables as $[R_1, R_2]$, to predict the outputs returns $EFF(10,157.7) = 103.4181$ (we all know the efficiency could not exceed 100. But, if the resulted decision variables cause the maximum efficiency experimentally, the total optimization result is satisfying) while performing an experiment with the same inputs results in 98.4652. On the other hand,

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the range of efficiency for the 25 initial experiments, which has been used to accomplish the regression, is [62 98.5003].

Since we need a trade off between two objectives and the final design will lead to some amounts different from the optimized point of each objectives separately, it seems the accuracy of fitted functions are satisfying. However, it is possible to reach more accuracy if we do more experiments, much more than 25 experiments, to use more data for the regression phase.

Now we are going to obtain a Pareto Front using gamultiobj for our non-linear quadratic two objectives two decision variables with linear inequalities upper/lower bounds problem. In a multi-objective problem, the designer should define the importance of each objective function. Suppose there is two functions and one of them are three times more important the other. It means the first function has a "weight" of %75 importance and the second function has the weight of %75 importance. Pareto Front gives a general sight to the designer to understand the situation of both functions at a same diagram when "weight" vector of importance is changed between %0 to %100 for each of them.

Figure 4 shows Pareto Front of the problem. It is observable that there is an optimal point in the middle of the diagram. At the right side of this point efficiency is higher but the other objective function, ground current or EMI, is getting worse rapidly. It is the same for the left side of the diagram when the performance of EMI is better but the efficiency is decreasing rapidly. If the designer has a constraint for one the function, it is also very easy to recognize the demanded weight factor by looking at Pareto Front. For example, if EMI should be under $56 \mu AdB$, the best possible efficiency will be %94. It is worthy to mention another time that the efficiency fitness function has error in comparison to reality. For this reason, there are some amounts more than 100 for the efficiency. But, there is no problem since those amounts returns higher efficiency, however below 100, in reality. The important point is the fitness function of efficiency models the behavior of the system very well.
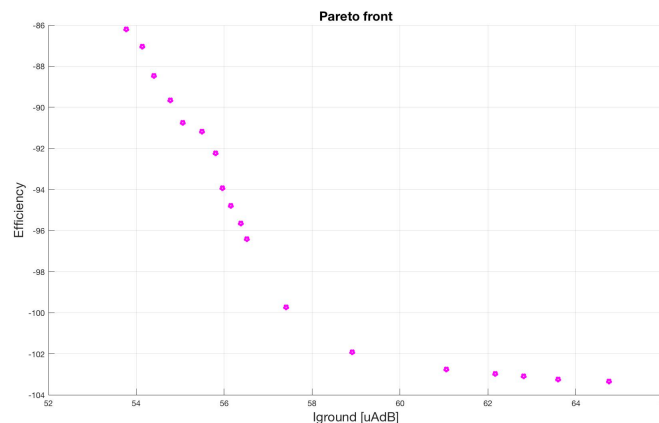

Figure 4: Pareto Front of the multi-objective optimization problem using genetic algorithm