

TEST FUNCTIONS FOR INTEGRAL APPROXIMATION

- 1D case.

Examples of integral approximations in the 1D case using the Jacobi weight function defined as $w(x) = (1-x)^\alpha(1+x)^\beta$.

Example 1:

$$\int_{-1}^1 e^x(x+5) w(x) dx$$

Example 2:

$$\int_{-1}^1 |\sin(1-x)|^{\frac{9}{2}} w(x) dx$$

Example 3:

$$\int_{-1}^1 x \left| \cos\left(\frac{1}{2}-x\right) \right|^{\frac{3}{2}} w(x) dx$$

Example 4:

$$\int_{-1}^1 |x-1|^{\frac{3}{2}} \sin(x) w(x) dx$$

The exact solution is computed with the Gaussian quadrature formula with 512 nodes.

- 2D case.

Examples of integral approximations in the 2D case using the Jacobi weight function defined as $w(x_1, x_2) = w_1(x_1)w_2(x_2) = (1-x_1)^{\alpha_1}(1+x_1)^{\beta_1}(1-x_2)^{\alpha_2}(1+x_2)^{\beta_2}$.

Example 1:

$$\int_{-1}^1 \int_{-1}^1 |\sin(1-x_1)|^{\frac{9}{2}} (1+x_1+x_2) w(x_1, x_2) dx_1 dx_2,$$

Example 2:

$$\int_{-1}^1 \int_{-1}^1 x_1 \left| \cos\left(\frac{1}{2}-x_1\right) \right|^{\frac{3}{2}} + x_2 |\sin(1+x_2)|^{\frac{3}{2}} w(x_1, x_2) dx_1 dx_2$$

Example :

$$\int_{-1}^1 \int_{-1}^1 e^{1+x_1+x_2} |x_1-1|^{\frac{7}{2}} w(x_1, x_2) dx_1 dx_2$$

The exact solution is computed with the Gaussian cubature formula with 512 nodes in both variables.