Boundary value problems with nonsmooth and multivalued terms

Vasile Staicu
University of Aveiro, Portugal

The aim of this course is to present some methods and tools used in the study of nonlinear boundary value problems involving multivalued maps and nonsmooth functions and illustrate those methods with some recent results concerning existence and multiplicity of solutions.

We start by introducing some basic aspects of smooth and non smooth calculus in Banach spaces, providing some basic tools to approach the subjects in the sections that follows.

Then we develop the critical points theory for smooths and non smooths functionals, the main tools in the variational method for solving boundary value problems, which consists of trying to find solutions for a given boundary value problem by looking for stationary points of a real functional defined on a space of functions in which the solution of the boundary value problem is to lie.

Degree theory is a basic tool of nonlinear analysis and produces powerful existence and multiplicity results for nonlinear boundary value problems. A special attention is dedicated to a generalization of Brouwer degree theory to multivalued perturbations of monotone type maps, developed in our joint paper with Aizicovici and Papageorgiou (Mem. Amer. Math. Soc., 196, 2008).

Then we determine some important spectral properties of the negative p-Laplacian with Dirichlet and Neumann boundary conditions.

We proceed with some basic notions and results from Morse theory that we use to produce nontrivial smooth solutions to boundary valued problems.

The above mentioned tools will be used to prove several existence and multiplicity results, with sign information for all the solutions of nonlinear boundary value problems driven by the Laplacian, the p-Laplacian and then try to extend such results to nonlinear boundary value problems driven by fractional order differential operators and to nonlocal pseudo-differential inclusions, possibly including obstacles or constraints. It is reasonable to expect that the pairing of nonlocal diffusion operators and set-valued reaction will provide a more realistic model for applications to such problems as quantum mechanics, image restoration, and financial mathematics, which typically present a high degree of uncertainty, rather than elliptic equations with smooth, single-valued reactions.

Lecture notes will be provided and open questions will be considered and analyzed with the Ph. D. students.

. . . .