Laboratory of
Numerical Algorithms for Engineering

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Euler's method with applications

Exercise 1 Write a script named "testMalthus" which computes the solution of the Malthus model

$$\begin{cases} y'(t) = (K - M)y(t), \quad t \in [0, T] \\ y(0) = y_0 \end{cases}$$

by using the Euler method. Here, K and M denote the birth and death rate of a population, respectively.

It is expected that a function file named "malthus" containing the generating function $f(t, y) = (K - M)y(t)$ is written.

Test the algorithm by using the exact solution $y(t) = y_0 e^{(K-M)t}$ and fixing $T = 1$, $y_0 = 1000$, $M = 0.4$ e $K = 0.8$.

Finally, display in a single graph the exact and the approximated solution in the case when $M = 0.4$ e $K = 0.2, 0.4, 0.8$.

Exercise 2 Write a function file named "Euler", which perform the Euler method for the solution of a Cauchy problem having m ODE.

Write a script named "testEuler" to apply the algorithm "Euler" the following problem

(a)

$$\begin{cases} y'(x) = -y(x) + \cos(x) - \sin(x), \quad x \in [0,\ 1] \\ y(0) = 2 \end{cases}$$

whose exact solution is $y(x) = e^{-x} + \cos(x)$.

(b)

$$\begin{cases} y''(x) = 3y'(x) - 2y(x), \quad x \in [0,\ 1] \\ y(0) = 1,\ y'(0) = 1 \end{cases}$$

whose exact solution is $y(x) = e^x$.

Exercise 3 Write a script named "testprey" that call back the function "Euler" for computing the solution of the Lotka-Volterra model

$$\begin{cases} F'(t) = \alpha F(t) - \beta P(t)F(t), \quad t \in [0, T] \\ P'(t) = \gamma P(t)F(t) - \delta P(t) \\ F(0) = F_0,\ P(0) = P_0. \end{cases}$$

where F is the number of prey, P is the number of predator, $\alpha, \beta, \gamma, \delta$ are positive real parameters describing the interaction of the two species.

Display in a graph the solution in the case when $T = 30$, $F_0 = 800$, $P_0 = 30$, $\delta = 1$, $\alpha = 0.25$, $\beta = 0.001$ e $\gamma = 0.001$. Finally, display the phase plot.

Exercise 4 Let us approximate the solution of the following SIR model (Susceptible, Infectious, Recovered)

$$\begin{cases} S'(t) = -\beta I(t)S(t), \quad t \in [0,T] \\ I'(t) = \beta I(t)S(t) - \gamma I(t) \\ R'(t) = \gamma I(t) \\ S(0) = S_0,\ I(0) = I_0,\ R(0) = R_0 \end{cases}$$

where β is the infection rate and γ is the cure rate.